Winning with Losses: The Surprising Success of Negative Strategies in Social Interaction Behavior

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In the vast network of social interactions and behaviors, it is common to find certain schemes that, on their own, might appear counterproductive to societal progress. When observed in isolation, these schemes often seem to hinder more than they help. However, due to society's complexity, the hidden potential of combining these seemingly detrimental schemes often goes unnoticed. Here, we investigate two such social behaviors, reputation and reciprocity, and their role in explaining Darwin's survival of the fittest, examining how these fundamental principles govern individual interactions and shape broader social dynamics. We outline the dynamics of these two social behaviors and underline the importance of combined strategies in enhancing group welfare and contributing to interdisciplinary research in social physics.

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With the advent of technology and the deep analytical capabilities of network science, researchers are now equipped to explore the intricate workings of sociodynamic processes more profoundly than ever before. Into this mix enters the burgeoning field of social physics, which seeks to apply principles and methods from physics to understand and predict human behavior in large groups [1]. One intriguing concept explored in this domain is Parrondo's paradox: combining or switching between two losing strategies might surprisingly achieve a winning outcome [2]. The role of Parrondo's paradox in complex systems has sparked key research into chaotic many-body [3], quantum [4], and algorithmic network applications [5], where combining elements yields opposing beneficial results. Similarly, social physicists aim to uncover hidden mechanisms that govern societal phenomena by integrating the paradox's counterintuitive principles. Concepts like collective voting and preference aggregation, traditionally viewed as purely sociological constructs, are now being dissected using the tools and frameworks of physics [6–8].

The enigma of human cooperation persists as a subject of scholarly interest, particularly when considering Darwin's principle of survival of the fittest, as evidenced by existing literature [9,10]. Current theories hint at two main facets of social interaction, reputation and reciprocity, as potential drivers behind this cooperative evolution [11,12]. Reputation revolves around building and sustaining trust, social worth, and overall community standing. Conversely, reciprocity governs the mutual exchange of actions or benefits, influencing our choices. These conclusions are derived from a sociological perspective and seek to interpret cooperation and altruism as forms of propagating behavior [13–15]. Intriguingly, by integrating these concepts with the principles underlying Parrondo's paradox, our objective is to elucidate the necessity for collective behavioral dynamics.

Here, we define a social group as individuals subjected to a network structure who are not necessarily mutually connected, but all the individuals can interact with an environment. This concept is likened to the small-world network, the primary network we will use in this study.

Reputation schemes are pivotal in promoting positive actions and ensuring compliance with agreements. While numerous reputation systems have been introduced in real-world scenarios or academic studies, this work adopts the approach termed the beta reputation system [16,17]. The beta reputation system is firmly grounded in statistical theory, providing a robust framework for feedback. The positive (success) and negative (failure) interactions accumulated by individual i at interaction t are counted through two numeric variables, s_i^t and f_i^t. Each successful interaction leads to an increment of s_i given by $s_i^\tau = s_i^t + 1$, $\tau > t$. Correspondingly, each failed interaction leads to an increment of f_i. The expected reputation score given by the number of positive and negative interactions is

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$$\mathbb{E}[\phi_i^t] = \frac{\alpha_i^t}{\alpha_i^t + \beta_i^t}, \qquad (1)$$

where parameters $\alpha_i^t = s_i^t + 1$ and $\beta_i^t = f_i^t + 1$ allow for positive and negative interactions to be mapped to the shape parameters of the beta distribution. Accounting for a mechanism to allow recent feedback to have a more pronounced effect on the reputation, thus reducing the contribution of older interactions, the values s and f can be modified as

$$s_i^{t+1} = \lambda_i s_i^t \quad \text{and} \quad f_i^{t+1} = \mu_i f_i^t, \qquad (2)$$

where $0 \le \lambda, \ \mu < 1$ are scaling factors that lead to a diminishing contribution of older interactions. It follows that $\phi_i^t \sim \Phi(x|\alpha_i^t, \beta_i^t)$, with probability density function defined by

$$\Phi(x|\alpha_i^t, \beta_i^t) = \frac{\Gamma(\alpha_i^t + \beta_i^t)}{\Gamma(\alpha_i^t)\Gamma(\beta_i^t)} x^{\alpha_i^t - 1}(1-x)^{\beta_i^t - 1}, \qquad (3)$$

where $\Gamma(\cdot)$ is the gamma function, with probability variable $x \neq 0$ if $\alpha < 1$, and $x \neq 1$ if $\beta < 1$.

Every individual operates within a network but is also part of a broader environment. This environment encompasses entities or clusters beyond the primary network, presenting opportunities or challenges based on the individual's reputation. A high expected reputation score $\mathbb{E}[\phi_i^t]$, signifying predominantly positive past interactions, prompts the environment to reward the individual. In contrast, a lower score results in a penalty. This can be a losing outcome if the environment punishes more than rewarding an individual.

On the other hand, reciprocity is about mutual exchange and balance. Borrowing from the principle of "observational reciprocity," this study investigates reciprocity as a phenomenon influenced by observed actions. Observational reciprocity posits that individuals do not act in isolation; their choices often reflect their surroundings and motivation for others to act. For example, if an individual, A, observes another individual giving, this elicits a response, compelling A to participate in a similar act of generosity. Consequently, motivated by the altruism witnessed, A chooses an adjacent beneficiary, B, to whom they may extend their donation. In gratitude, B endorses A, resulting in a positive recommendation, increasing the positive interaction count s_A. However, if A chooses not to act on the observation to a potential receiver, the negative interaction count f_A is increased.

Time-decaying weights can be incorporated to enhance observational reciprocity and its relevance to real-world interactions to put a time dependence on an observer's motivation to give. Consider two situations (1) A has just received a unit of capital, or (2) A has just observed another individual giving. A's motivation to give in these two instances is the highest, with motivation decreasing with increasing time in both cases. Thus, the tendency to "pay it forward" can be modeled with a time-decaying weight, as characterized by the Ebbinghaus forgetting curve [18],

TABLE I. Table of parameters.

Variable	Description	Initialization
s_i	Success value	0
f_i	Failure value	0
λ_i	Scaling factor for s_i	$U[0.8, 1.0)$[a]
μ_i	Scaling factor for f_i	$U[\lambda_i, 1.0)$[a]
g_i	Boolean indicator to track giving in a reciprocity interaction	{0,1}
τ_i	Time step when i last received in a reciprocity interaction	0
t_i'	Time step when i last gave in a reciprocity interaction	0
$c_i(t)$	Capital as a function of time steps	0

[a] $U[a, b)$ denotes the uniform distribution in the interval $[a, b)$.

$$w(\Delta t) = \frac{100 k_1}{[\log(\Delta t)]^{k_2} + k_1}. \qquad (4)$$

where $k_1 = 1.84$ and $k_2 = 1.25$ are constants fitted from Ebbinghaus's work and verified in later studies [19]. This model of motivation, influenced by observational and pay-it-forward reciprocity, reflects real-world social, emotional, and psychological factors. While observational reciprocity underscores the role of mutual social influence [20,21], pay-it-forward reciprocity reveals a more transient nature of cooperation driven by inherent prosocial behavior [22].

Let $N = \{1, 2, \ldots, n\}$ be a set of individuals in a social group. Each $i \in N$ is a node in the network connected to a subset of individuals $M_i \subseteq N \backslash \{i\}$ by undirected edges E_i. We denote the social group as (N, E), where E are all the edges in the network. Consequently, $(M_i, E_i) \cup \{i\}$ is a subset of (N, E) containing i, all neighbors of i, and all edges $(i, j) \in E_i$. For each $i \in N$, we keep track of the terms in Table I.

s_i and f_i increase incrementally for every positive and negative interaction, respectively. λ_i and μ_i remain fixed after initialization. In our setup, $\mu \ge \lambda$ is in concurrence with negativity bias in sociology, where individuals tend to remember negative experiences or give more weight to negative comments than positive ones. g_i is an indicator that tracks whether i gave in a reciprocity interaction. If i gave, then $g_i = 1$; otherwise, $g_i = 0$.

The game-theoretic Parrondo's paradox emerges through multiple iterations of these interactions typically defined over a period T, the number of times the game is played, and K, the number of total games averaged over the long term. To determine what "winning" or "losing" is, we inspect the average fitness index of the entire network. Consequently, the average fitness index for the entire social group, taking into account the diverse and dynamic nature of real-world social interactions, is given by

$$\bar{d}(t) = \frac{\sum_i c_i(t)}{|N|KT}, \tag{5}$$

where $|N|$ is the size of the group. We set $c_i(0) = 0, \ \forall \ i$ as we are only interested in the net fitness change when investigating Parrondo's effect. One can envision scenarios where Parrondo's effect will emerge when all nodes in the network adopt either reputation-only or reciprocity-only strategies, leading to a negative average fitness index for the entire social group, yet strategically switching between adopting reputation or reciprocity schemes, which might lead to a positive $\bar{d}$.

At each interaction instance t, a randomly selected individual i from the network is chosen. For the reputation scheme, the environment interacts with the randomly selected individual i according to the evidence of good reputation. Suppose the selected individual i has positive and negative recommendations following the descriptions of the beta reputation model. Then, i gains a unit of capital from the environment if it is judged to have a high expected reputation score, given by Eq. (1). As this judgment might be relative, the expected reputation score is judged against a random value from the interval [0, 1). Thus, if $r < \mathbb{E}[\phi_i^t]$, for $r \sim [0, 1)$, then i gains a capital. Conversely, if it is adjudged to have a low expected reputation score, the environment punishes it by taking away two units of capital. The reputation mechanism is described in Algorithm 1 of the Supplemental Material [23].

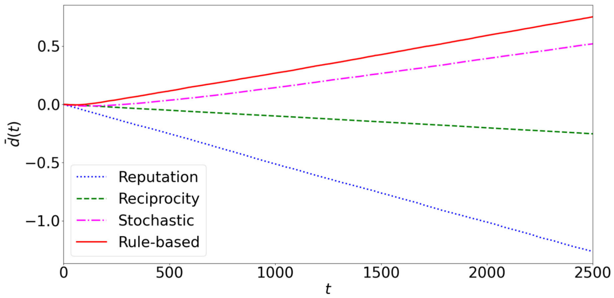
Similarly, under the rules of reciprocity, at each interaction instance t, a randomly selected individual i from the network is chosen as the principal, and a receiving individual j is chosen from the subset of nodes M_i. In every reciprocity interaction, i must give away a unit of capital to another individual or, failing which, then to the environment. Suppose in some previous interaction instances t', individual l was the principal, observational reciprocity dictates that if $l \in M_i$, then i may give, subject to two different time intervals: (1) The interval since it last observes a neighbor giving: i gives if it observes l giving in a recent past, that is for a random value from the interval [0, 1), if $p < w(t - t_l')$, for $p \sim [0, 1)$, and (2) The interval since it last received: i gives if it tends to pay it forward, that is, for a random value from the interval [0, 1), if $q < w(t - \tau_i)$, for $q \sim [0, 1)$. If these two conditions and the condition for observational reciprocity are fulfilled, then i will give a unit of capital to j. j being a benefactor of i, will give i a positive recommendation, adding to i's success value. If the conditions are not met, i donates a unit of capital to the environment. j, having not received from i, will give a negative recommendation, adding to i's failure value. The reciprocity scheme is described in Algorithm 2 of the appendix.

A naive observation might conclude that in either scheme the chance of individuals losing to the environment is higher than gaining from the environment. For the reputation scheme, one is rewarded with a singular capital from the environment but is punished with two. Similarly, the reciprocity scheme only allows for the redistribution of capital or loss of capital. In reality, diverse schemes can be adopted by different individuals. Thus, we suggest two forms of switching: (1) stochastic switching, where the individual randomly selects one of two schemes to employ with equal probability, and (2) rule-based switching, where the individual only selects the reputation scheme if it passes the reputation threshold ρ; otherwise, it employs the reciprocity scheme.

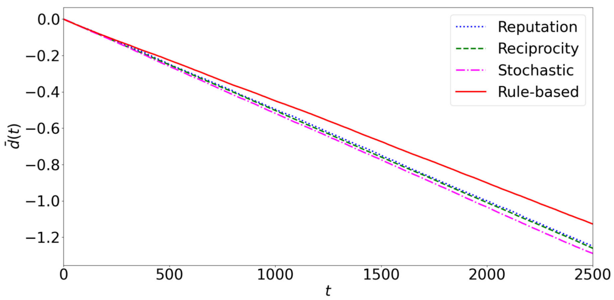
Since the average fitness $\bar{d}$ is a time-varying function of t and c_i is nonlinear, this makes analytic results harder to come by, and simulation necessary. Thus, we study the dynamics of Parrondo's paradox through simulations in various network contexts. The three networks we consider are the small-world, scale-free, and Erdős-Rényi random networks. The simulations are performed via PYTHON3.10.9, using NetworkX3.1 to generate and perform manipulations on the network. A small-world network is generated using the built-in function `connected_watts_strogatz_graph(n,k,p)` according to the algorithm proposed by Watts and Strogatz [24]. The scale-free network is generated by using the function `barabasi_albert_graph(n,m)`, and the random network using `erdos_renyi_graph(n,p)`.

Figure 1 shows the results generated with the network generating functions over an interaction period of $T = 2500$, averaged over $K = 10^5$ such experiments, with the threshold set at $\rho = 0.8$. Drawing on principles from statistical mechanics at the micro level, individuals may exhibit suboptimal behaviors influenced by immediate concerns of reputation and reciprocity. However, macroscopic benefits are revealed when these individual actions are aggregated at a network level. Figure 1(a) shows that there is indeed a scenario where the reputation and reciprocity schemes are individually losing, but the stochastic switching and rule-based switching give a winning outcome. Empirical results for the small-world network reveal that the reputation and reciprocity schemes are individually losing, while stochastically switching between these schemes with equal probability returns a net positive outcome. Furthermore, for the rule-based switching, we even achieve a better outcome. Therefore, in our subsequent discussion we discuss only the case for small-world networks; the other network topologies are included in the Supplemental Material [23].

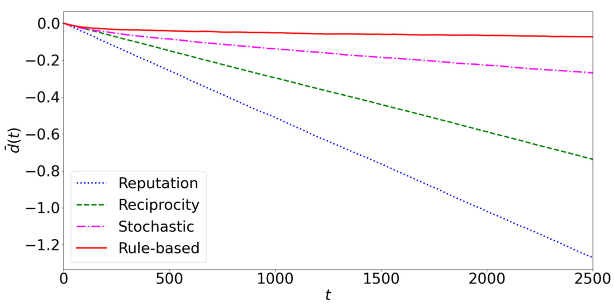
Effects of group size $|N|$—Comprehending the dynamics of reputation and reciprocity is crucial for unraveling the intricacies of complex social networks, with profound implications for real-world applications. A key element in these networks, which our findings illuminate, is the size



(a) Small-world network:
`connected_watts_strogatz_graph(n=20,k=n-2,p=0.5)`



(b) Scale-free network:
`barabasi_albert_graph(n=20,m=int(n/2))`



(c) Erdős-Rényi random network:
`erdos_renyi_graph(n,0.75)`

FIG. 1. Parrondo's effect is achieved in the specific case for the small-world network, where $|N| = 20$, $T = 2500$, $K = 10^5$, and the rule-based switching is imposed for $\rho = 0.8$. Parrondo's paradox is not observed for the scale-free and random networks.

or the total number of individuals within a social group, a factor that has significant bearings on various societal and organizational contexts. By investigating the relationship between $\bar{d}$ and $|N|$, we aim to decipher how the overall fitness of a network is influenced by its size. Such insights can offer valuable information on optimal network sizes for maximum fitness or the effects of overcrowding in dense networks. We plot the average fitness of the group at $t = 2500$ for a range of group sizes. In particular, we consider small-world networks with the following properties: `connected_watts_strogatz_graph (n, k = n − 2, p = 0.5)`, for $n \in [10, 500]$.

Figure 2 reveals the relationship between the nodal size of a small-world network and the average fitness index of the group. In particular, we observed for group sizes $|N| \geq 10$, Parrondo's effect is always observed. However, it is worth noting that with increasing group sizes $|N| \to \infty$, Parrondo's effect gets significantly weaker. Of interest is the domain $|N| \in (14, 22)$ for which we see the peak average fitness. This corresponds to a particular group size proposed in Dunbar's layers. Dunbar suggested different layers or circles of intimacy, one of which is the layer associated with "good friends." These 15 or so persons with whom an individual is close to are those for whom one might go out of their way to help [25,26]. Analyses of friendship networks on social media platforms suggest that even with digital tools, people maintain a similar number of meaningful relationships [27,28]. Previous research on Parrondo's paradox in synchronized decision making within social networks indicates an optimal group size [29]. Similarly, this work uses Parrondo's paradox, albeit in the context of reputation and reciprocity, leading to a different optimal group size. This alignment of group sizes with the stratified layers identified in Dunbar's theory underscores the theory's applicability in diverse social contexts.

Effects of switching threshold ρ—The pivotal parameter that governs the rule-based switching between the reputation and reciprocity schemes is the reputation threshold ρ. By exploring the relationship between $\bar{d}$ and ρ, our goal is to understand how the range of thresholds can impact the average fitness of the network. This is instrumental in optimizing rule-based systems, ensuring that they are calibrated to thresholds that maximize the capital of the network. We plot the average fitness of the group at $t = 2500$ for a fixed group size of $|N| = 20$ with node degree $|N| - 2$. In particular, we consider small-world networks with the following property: `connected_watts_strogatz_graph(n = 20,k = 18,p = 0.5)`, varying $\rho \in [0.0, 1.0]$. Since ρ is a threshold imposed only on the rule-based switching, it is clear that ρ does not affect the outcome of both the reputation and reciprocity schemes individually, and they are both losing, as seen in Fig. 2. Thus, we present the results of the average fitness index for the rule-based switching only.

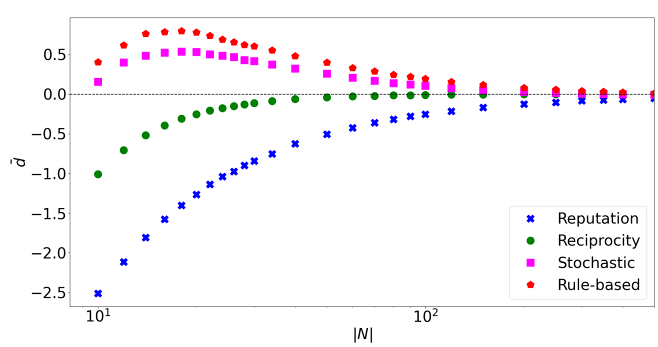


FIG. 2. Plot of the average fitness index of the group, for a small-world network, against the population size for $T = 2500$ over $K = 10^5$ experiments, where the rule-based switching is imposed for $\rho = 0.8$. The peak is achieved at $|N| = 20$. Parrondo's paradox is not observed for $|N| < 10$, and $|N| \geq 500$ for the stochastic scheme.

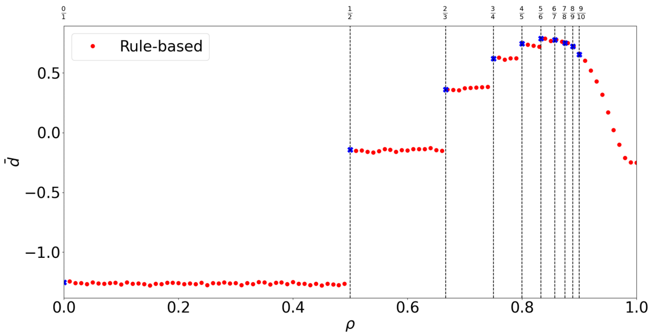


FIG. 3. Plot of the average fitness index of the group, for a small-world network, against the rule-based threshold for $T = 2500$ over $K = 10^5$ experiments, where the group size is $|N| = 20$. Parrondo's effect is observed for $\rho \in [2/3, 25/26]$. Changes in $\bar{d}$ are observed at $\rho_x = [x/(x+1)]$; the blue crosses mark $\bar{d}$ for the first ten terms of ρ_x.

Intuitively, setting a higher threshold ρ implies that individuals with low expected reputation scores will not employ the reputation scheme, lowering the chance of punishment from the environment. On the other hand, there is a maximum value for ρ, after which we will not have Parrondo's paradox as it is akin to every individual employing the reciprocity scheme, which is losing. What is not immediately clear is the relationship between ρ and $\bar{d}$. From our simulation results, Fig. 3 reveals that Parrondo's paradox, or a winning outcome, is observed in the domain $\rho \in [2/3, 25/26]$. However, a fascinating nonlinear behavior is observed in the change of $\bar{d}$, which occurs at various points ρ_x in the domain of ρ satisfying

$$\rho_x = \frac{x}{x+1}, \qquad x \in \mathbb{N}_0. \tag{6}$$

This behavior is peculiar as it does not reveal the exact value of $\bar{d}$ but indicates when to expect a change in $\bar{d}$. But more importantly, this suggests a phase transition at the tipping point. In particular, as $x \to \infty$, the term $\rho_x - \rho_{x-1} \to 0$ suggests a transition from discrete intervals to continuous change in the average fitness index. This is likened to studies in the field of social sciences, especially in the study of behavioral tipping points and payoffs in network cascades [30,31], which are worth studying in greater depth. This observation underscores the profound capability of rule-based switching mechanisms inherent in Parrondo's paradox to emulate and forecast key aspects of real-world social phenomena. Such insights are invaluable for developing sophisticated models and strategies in various fields, ranging from social sciences to policy making, where accurate predictions of social behavior and dynamics are crucial.

We also performed simulations on other network topologies recorded in the Supplemental Material [23]. Based on our observation, Parrondo's paradox is strongly observed in small-world networks, weakly in the Erdős-Rényi network, and absent in scale-free networks. We postulate that the small-world property, where a combination of high clustering and short average path lengths, facilitates short-term observational reciprocity and accumulation of reputation, hence fostering cooperative behaviors and enhancing group welfare.

In the complex arena of human social interactions, the interplay of reputation and reciprocity emerges as fundamental in orchestrating collective outcomes. These dynamics, deeply embedded in the fabric of human behavior, are instrumental in guiding and influencing the course of social structures and relationships, thereby playing a critical role in determining the efficacy and nature of group interactions and societal progress.

Revisiting the previously raised question regarding the manifestation of altruism and cooperation within the framework of "survival of the fittest," it becomes clear that these social behaviors are not just ancillary but central to understanding the evolution and dynamics of collective success. Perhaps reputation and reciprocity are fair or losing outcomes, which explains why individuals do not employ any of these schemes in isolation to "survive". Rather, it is the interplay of these two schemes that results in beneficial outcomes. In such a case, it is more "survival of the fittest social group" that thrives, and not so much the fittest individual. In traversing the complex network structures of human social systems, the synergistic insights gleaned from Parrondo's paradox provide an invaluable perspective for decoding social dynamics. This unique approach effectively merges the analytical frameworks of game theory and anthropology, offering profound implications for understanding human behavior.

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Data availability—Codes are made available at Ref. [32].

[1] M. Jusup, P. Holme, K. Kanazawa, M. Takayasu, I. Romić, Z. Wang, S. Geček, T. Lipić, B. Podobnik, L. Wang, W. Luo, T. Klanjšček, J. Fan, S. Boccaletti, and M. Perc, Social physics, Phys. Rep. **948**, 1 (2022).

[2] G. P. Harmer and D. Abbott, Losing strategies can win by Parrondo's paradox, Nature (London) **402**, 864 (1999).

[3] J. Buceta, K. Lindenberg, and J. M. R. Parrondo, Stationary and oscillatory spatial patterns induced by global periodic switching, Phys. Rev. Lett. **88**, 024103 (2001).

[4] A. Panda and C. Benjamin, Order from chaos in quantum walks on cyclic graphs, Phys. Rev. A **104**, 012204 (2021).

[5] C. Benjamin and N. Dudhe, Resolving degeneracies in Google search via quantum stochastic walks, J. Stat. Mech. (2024) 013402.

[6] J. W. Lai and K. H. Cheong, Social dynamics and Parrondo's paradox: A narrative review, Nonlinear Dyn. **101**, 1 (2020).

[7] J. W. Lai and K. H. Cheong, A comprehensive framework for preference aggregation Parrondo's paradox, Chaos **32**, 103107 (2022).

[8] J. W. Lai and K. H. Cheong, Evaluation of single-prioritization voting systems in controlled collective Parrondo's games, Nonlinear Dyn. **107**, 2965 (2022).

[9] E. Pennisi, How did cooperative behavior evolve?, Science **309**, 93 (2005).

[10] M. A. Nowak, Five rules for the evolution of cooperation, Science **314**, 1560 (2006).

[11] C. Xia, J. Wang, M. Perc, and Z. Wang, Reputation and reciprocity, Phys. Life Rev. **46**, 8 (2023).

[12] J. W. Lai and K. H. Cheong, A Parrondo paradoxical interplay of reciprocity and reputation in social dynamics, Chaos Solitons Fractals **179**, 114386 (2024).

[13] E. Fehr and U. Fischbacher, The nature of human altruism, Nature (London) **425**, 785 (2003).

[14] S. Aral, L. Muchnik, and A. Sundararajan, Distinguishing influence-based contagion from homophily-driven diffusion in dynamic networks, Proc. Natl. Acad. Sci. U.S.A. **106**, 21544 (2009).

[15] D. G. Rand and M. A. Nowak, Human cooperation, Trends Cognit. Sci. **17**, 413 (2013).

[16] A. Jøsang and R. Ismail, The beta reputation system, in *Proceedings of the 15th BLED Electronic Commerce Conference e-Reality: Constructing the e-Economy* (AIS eLibrary, Bled, Slovenia, 2002), Vol. 41.

[17] X.-L. Liu and S.-W. Jia, An iterative reputation ranking method via the beta probability distribution, IEEE Access **7**, 540 (2019).

[18] H. Ebbinghaus, Memory: A contribution to experimental psychology, Annu. Rev. Neurosci. **20**, 155 (2013).

[19] J. M. J. Murre and J. Dros, Replication and analysis of Ebbinghaus' forgetting curve, PLoS One **10**, e0120644 (2015).

[20] A. Mahmoodi, B. Bahrami, and C. Mehring, Reciprocity of social influence, Nat. Commun. **9**, 2474 (2018).

[21] M. Wörle, S. Essler, and M. Paulus, Paying it back and forward: The impact of experiencing and observing others' sharing and stinginess on preschoolers' own sharing behavior and expectations, J. Exp. Child Psychol. **198**, 104886 (2020).

[22] Y. Horita, M. Takezawa, T. Kinjo, Y. Nakawake, and N. Masuda, Transient nature of cooperation by pay-it-forward reciprocity, Sci. Rep. **6**, 19471 (2016).

[23] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.133.167401 for analytical derivations, algorithms, and supplementary results.

[24] D. J. Watts and S. H. Strogatz, Collective dynamics of 'small-world' networks, Nature (London) **393**, 440 (1998).

[25] R. Dunbar, Neocortex size as a constraint on group size in primates, J. Hum. Evol. **22**, 469 (1992).

[26] W.-X. Zhou, D. Sornette, R. A. Hill, and R. I. M. Dunbar, Discrete hierarchical organization of social group sizes, Proc. R Soc. B **272**, 439 (2005).

[27] P. M. Carron, K. Kaski, and R. Dunbar, Calling Dunbar's numbers, Soc. Networks **47**, 151 (2016).

[28] B. Gonçalves, N. Perra, and A. Vespignani, Modeling users' activity on twitter networks: Validation of Dunbar's number, PLoS One **6**, e22656 (2011).

[29] J. W. Lai and K. H. Cheong, Boosting Brownian-inspired games with network synchronization, Chaos Solitons Fractals **168**, 113136 (2023).

[30] C. Bicchieri, *The Grammar of Society: The Nature and Dynamics of Social Norms* (Cambridge University Press, Cambridge, England, 2005).

[31] D. J. Watts, A simple model of global cascades on random networks, Proc. Natl. Acad. Sci. U.S.A. **99**, 5766 (2002).

[32] https://github.com/joellaiwj/reputation-reciprocity-pp/