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A Parrondo paradoxical interplay of reciprocity and reputation in social dynamics

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ABSTRACT

Our study investigates the role of reciprocity and reputation in shaping social dynamics within networks. We uncover this by creating a model network with agents who follow unique beliefs and rules. While relying solely on either reciprocity or reputation often leads to negative outcomes for a group, combining these strategies leads to unexpectedly positive results. This finding, akin to the counterintuitive Parrondo's paradox, illustrates the hidden potential of integrating different social strategies in boosting group welfare. This work has implications for understanding and leveraging the physics of social dynamics, such as understanding the 'Goldilocks domain' for different population sizes and Burt's theory for structural holes. This article sheds light on the nuanced interrelations between reciprocity and reputation and emphasizes their impact on social welfare, offering valuable insights for taking the first steps in enhancing collective welfare in social networks.

1. Introduction

Parrondo's paradox posits that two losing strategies, when alternated or combined in a certain manner, can unexpectedly produce a winning outcome [1,2]. Although traditionally explored within the realms of statistical mechanics and in coin-toss probability games [3–5], researchers have recently begun to extend its principles to varied disciplines, including economics [6], biology and ecology [7–9], and quantum systems [10,11]. Applications in multiple fields, including physical and biological systems, sparked the expansion of research into Parrondo's paradoxical phenomena away from traditional game theory to transdisciplinary systems science. Complex computational modeling combined with network science has been extensively employed to investigate sociodynamical processes, offering insights into real-world interactions and social organizations. In the same way, the burgeoning field of social dynamics, or 'social physics', through the novel lens of Parrondo's paradox offers a means to explain the intricate interplays within societal interactions [12,13] like collective voting, preference aggregation, and synchronization [14,15].

Central to our exploration in this article is to provide an explanation for the emergence of altruism and cooperation through the lens of game theory and social physics [16–19]. The question of how humans evolve to cooperate from the perspective of Darwin's survival of the fittest is in itself a paradox [20,21]. A recent review postulates that this could be due to two types of social interactions: reciprocity and reputation [22]. Reciprocity focuses on exchanging actions, benefits, or favors that underpin social exchanges [23]. Through the lens of reciprocity, it guides decisions to give and take, to cooperate or compete [24,25]. On the other hand, reputation studies how trust, perceived value, and social capital are acquired, nurtured, and maintained [25]. It provides insights into the rebalancing and evolution of individual actions [26,27]. By integrating these ideas with Parrondo's paradox, we aim to unveil new layers of complexity in understanding collective behaviors [28]. This article pioneers the integration of reciprocity and reputation within the context of Parrondo's games, offering unprecedented insights into their impact on social behavior and theories.

2. Methodology

Reciprocity is a foundational tenet of social behavior, dictating the exchange dynamics between agents in social systems [29]. The act of reciprocation, while seemingly straightforward, can manifest through various mechanisms. We delineate three primary methods of reciprocity execution: pay-it-forward, benefactor-driven, and observational reciprocity. These are illustrated in Fig. 1. In the **pay-it-forward reciprocity** mode, Agent A's decision to give is predicated on a previous receipt from another agent. When Agent A has been a beneficiary of a prior act of giving, this prompts Agent A to 'pay it forward' to Agent B.

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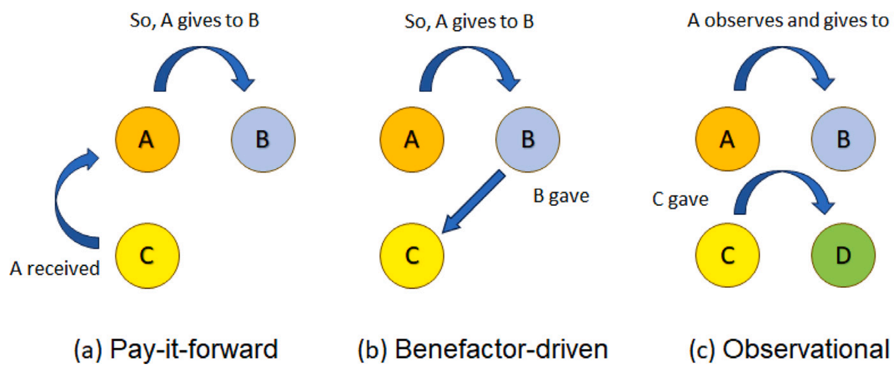

Fig. 1. The social behavior 'Reciprocity' follows the principle of propagating goodwill. The three forms of reciprocity are: (a) pay-it-forward, (b) benefactor-driven, and (c) observational reciprocity

This act of passing on the benefit embodies the essence of reciprocity. For **benefactor-driven reciprocity**, the reciprocal behavior is driven purely by the behavior of Agent B as a giver. Specifically, suppose Agent B had previously extended an act of giving toward another agent. In that case, Agent A responds in kind by reciprocating the gesture to Agent B. As for **observational reciprocity,** unlike the first two methods, which are rooted in direct experiences of Agent A and B, respectively, observational reciprocity is driven by third-party actions. In this scenario, if Agent A witnesses another agent engaging in an act or giving, it induces Agent A to reciprocate by giving to Agent B, even if this giving action was not directed towards Agent A. The underlying principle is the propagation of goodwill observed in the social group. Consequently, since Agent B receives a positive action from Agent A in all three cases, Agent B forms a positive recommendation for Agent A.

Reputation is a paramount attribute guiding agent–environment interactions. Specifically, the behavior of agents in response to their environment is often conditioned upon their perceived reputation. In our framework, the reputation of Agent A is adjudicated by its recommendation history, which records the two most recent interactions with other agents. The recommendation history is either positive (indicating favorable interactions), negative (indicating unfavorable interactions), or mixed (indicating a combination of favorable and unfavorable interactions). The determination of rewards for an agent by the environment is rooted in the Dempster–Shafer theory, using a mathematical approach to deal with evidence to evaluate the recommendation history [30–33].

The environment interacts with the agent based on the described positive, negative, or mixed recommendation history. For **positive reputation**, when Agent A's recommendation history is both positive, the environment infers that Agent A is plausibly deserving of a reward, which is the supporting evidence with uncertainty. In contrast, for **negative reputation**, if Agent A's recommendation history exclusively comprises negative interactions, there is a fuzziness on the reputation of Agent A. Yet, the environment still accords a reward, albeit with uncertainty. This is a counter-intuitive design in the system to incentivize change in behavior. Lastly, in the case of **mixed reputation**, where Agent A's recommendation history is a blend of positive and negative interactions, the environment rewards according to evidence against, with some uncertainty. In all three cases, the decision to confer a reward is contingent upon whether the probability of the decision aligns with the intervals of belief and plausibility delineated by the Dempster–Shafer theory, as seen in Fig. 2.

We proceed to delineate the algorithmic framework for the application of Parrondo's paradox to the two governing social rules. Let $N = \{1, 2, \ldots, n\}$ be a set of agents. Each agent $i \in N$ is a node in the network and is connected to a subset of agents $M^{(i)} \subseteq N\backslash\{i\}$ by undirected edges $e^{(i)}$. Each agent i in the network is further assigned a level of belief $\mathrm{Bel}(i) \sim U(0,1)$ and a level of plausibility $\mathrm{Pl}(i) \sim U(\mathrm{Bel}(i), 1)$. We initialize a matrix of recommendations, which keeps track of the recommenders for each agent i, and randomly assign two recommenders to each agent from the subset $M^{(i)}$. The recommendations are all set to be negative, i.e. $\mathrm{Recommend}(j,i) = 0$ as we assume that the agents are unfamiliar with each other at the beginning and thus unable to give positive recommendations. Lastly, we randomly assign $\mathrm{R}(i)$ and $\mathrm{G}(i)$, where $R, G \in \{0,1\}$. Here, R(i) indicates whether agent i has received from another agent during an interaction, and $G(i)$ denotes if agent i has given. The impact of social rules is quantified using a proposed metric termed "capital", where each agent's capital is $c^{(i)}$. The effect can be tracked over the number of interactions, t, giving $c^{(i)}(t)$.

At each interaction instance t, a randomly selected agent i from the network is chosen as the principal agent, and a receiving agent j is chosen from the subset of nodes $M^{(i)}$. Under the rules of reciprocity, i then randomly chooses to adopt one of the three reciprocity methods, intending to give one unit of capital. If i meets the initiating requirement, it proceeds to do so, noting that it has given a unit of capital to another agent (i.e., $\mathrm{G}(i) = 1$), j gains a unit of capital and notes that it has received from j (i.e., $\mathrm{R}(j) = 1$), giving i a positive recommendation. If i does not meet the initiating requirements, it gives the unit of capital away to the environment. j notes that it did not receive from i and gives a negative recommendation to i in this case. The reciprocity scheme is described in Algorithm 1.

Algorithm 1 Reciprocity scheme

Require: $i \in (N, e),\ j \in (M^{(i)}, e^{(i)})$

Randomly select one of three reciprocity models: (a) Pay-it-forward, (b) Benefactor-driven, (c) Observational. The initiating requirements are (a) $R(i) = 1$, (b) $G(j) = 1$, and (c) $G(k) = 1$, respectively. Where k is agent i of the previous interaction step.
if initiating requirement met **then**
    $c^{(i)} \leftarrow c^{(i)} - 1$
    $c^{(j)} \leftarrow c^{(j)} + 1$
    $G(i) \leftarrow 1$
    $R(j) \leftarrow 1$
    $\mathrm{Recommend}(j,i) \leftarrow 0$
else
    $c^{(i)} \leftarrow c^{(i)} - 1$
    $R(j) \leftarrow 0$
    $\mathrm{Recommend}(j,i) \leftarrow 0$
end if
In all instances, j becomes new recommender for i.

Similarly, under the rules of reputation, the environment interacts with the randomly selected agent i according to the evidence of good reputation. Suppose the selected agent i has positive recommendations. In that case, it gains a unit of capital from the environment based on the evidence supporting the belief up to a level of uncertainty (i.e., $\mathrm{Pl}(E^{(i)})$); otherwise, it loses a unit of capital to the environment. If, however, the agent has negative recommendations, it still gains a unit of capital, this time based on the level of uncertainty in the evidence (i.e., $\mathrm{Pl}(E^{(i)}) - \mathrm{Bel}(E^{(i)})$). If the agent has mixed recommendations, it

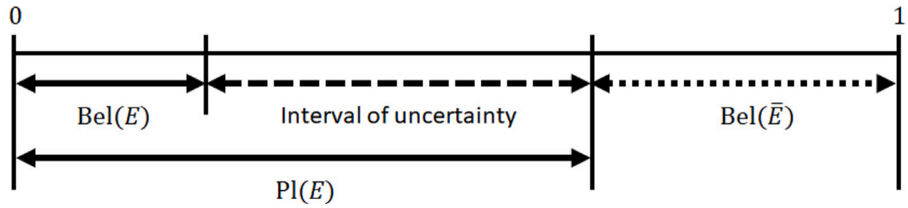

Fig. 2. Environment interaction with an agent by evaluating the belief-plausibility interval framework from Dempster–Shafer evidence theory. E is the evidence for a hypothesis, while $\bar{E}$ is evidence for the anti-hypothesis. Bel(E) is the interval of belief, Pl(E) is the interval of plausibility.

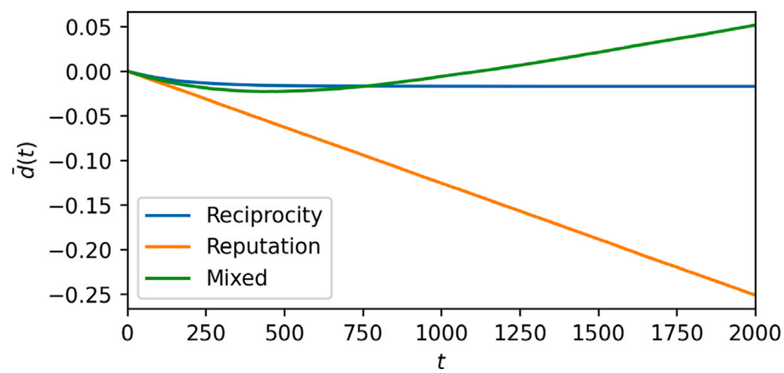

Fig. 3. The average fitness index of the population $\bar{d}(t)$ plotted against interactions t. Stochastically choosing (termed "mixed") between the reciprocity and reputation schemes with equal probability leads to Parrondo's paradox, where both reciprocity and reputation schemes employed independently are "losing" (negative average fitness), while the mixed scheme is "winning" (positive average fitness). The same simulation for interactions up to $t = 10^4$ is found in the Appendix.

will gain a unit of capital based on the evidence against the belief, up to a level of uncertainty. (i.e. $1 - \mathrm{Bel}(E^{(i)})$). The reputation scheme is described in Algorithm 2.

Algorithm 2 Reputation scheme

Require: $i \in (N, e)$, $j, k \in (M^{(i)}, e^{(i)})$, where j, k are the last two recommenders of i, p, the probability of occurrence.

```
if Recommend(j,i) and Recommend(k,i) = 1 then
    if p < Pl(i) then
        c^(i) ← c^(i) + 1
    else
        c^(i) ← c^(i) − 1
    end if
else if Recommend(j,i) and Recommend(k,i) = 0 then
    if p ≥ Bel(i) and p < Pl(i) then
        c^(i) ← c^(i) + 1
    else
        c^(i) ← c^(i) − 1
    end if
else
    if p ≥ Bel(i) then
        c^(i) ← c^(i) + 1
    else
        c^(i) ← c^(i) − 1
    end if
end if
```

In the game-theoretic Parrondo's paradox, we typically define T, the number of times the game is played, and K, the number of total games averaged over in the long term. To determine what "winning" or "losing" is, we inspect the average fitness index of the entire network. The average fitness index, at interaction t, of a single agent is defined as

$$\bar{d}(t) = \frac{w^{(i)}(t)}{KT}, \tag{1}$$

where $w^{(i)}(t) = c^{(i)}(t) - c^{(i)}(0)$, $w^{(i)}(t)$ and $c^{(i)}(t)$ are the winnings (i.e. change in the capital) and capital at interaction t of agent i, respectively. $c^{(i)}(0)$ is the original capital of agent i, and group size $|N|$. Without loss of generality, we can set $c^{(i)}(0) = 0$, $\forall i$ as we are only interested in the net change of fitness when investigating the Parrondo's effect. Thus, the average fitness index of the population can be simplified to give

$$\bar{d}(t) = \frac{\sum_i c^{(i)}(t)}{|N|KT}. \tag{2}$$

3. Results & discussion

We observe the evolution of the average fitness with the following parameters: $|N| = 100$, $T = 2000$, $K = 10^6$. We assume that (N, e) is the complete network for this part of the results and discussion. We observe how the average fitness of a highly connected population of size $|N|$ changes for 2000 interactions, averaged over 10^6 independent simulations. The result of the average fitness index of the population for the reciprocity, reputation, and mixed schemes is found in Fig. 3. Here, the mixed scheme randomly selects whether to adopt the reciprocity or reputation scheme at each interaction t with equal probability.

In Fig. 3, we observe that while the average fitness index of the population for each scheme starts off as negative, the average fitness index of the population for the mixed scheme contains a critical minimum point, leading to the emergence of Parrondo's paradox within the studied domain T. In statistical physics, phase transitions demonstrate how individual actions can give rise to collective behavior. Similarly, while individual agents in a group might not show structured behavior, their interactions can produce larger consistent patterns. Consequently, Parrondo's paradox does not emerge until a critical interaction value t_c, as shown. In the case of complete networks, subsequent simulations further reveal that this critical value depends on the population size $|N|$. This observation aligns with existing research, which indicates that Parrondo's paradox manifests within a defined parameter space–referred to as the 'Goldilocks domain'–where optimal conditions transform two individually losing strategies into a collectively winning one. One should note that as the network topology changes, this critical value t_c may depend on network parameters. Regardless, in the context of our study, we want to find the critical value that defines this zone for varying population sizes. In particular, for a population size $|N|$, we

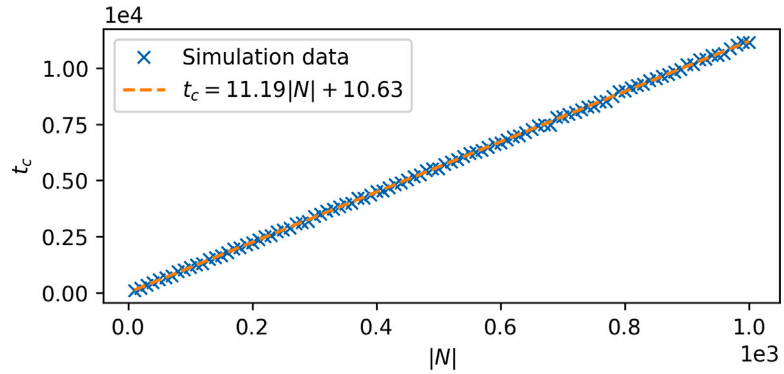

Fig. 4. The critical value t_c plotted against population size $|N|$. Parrondo's paradox does not emerge until t_c, and differs with $|N|$. Through regression, we found the relationship to be linear, given by $t_c = 11.19|N| + 10.63$.

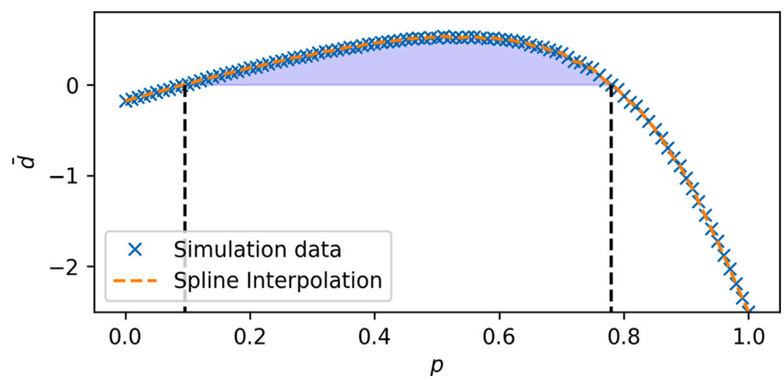

Fig. 5. The average fitness index of the population $\bar{d}$ as a proportion of reputation interactions p in the mixed scheme. If the population adopts a high proportion of any single scheme, it is collectively destructive, while mixed strategies can lead to favorable outcomes for $0.0951 \lesssim p \lesssim 0.7793$, with the optimal outcome achieved at $p \approx 0.4906$.

want to find the critical value t_c such that Parrondo's paradox emerges, that is we go from

$$\begin{aligned} &\bar{d}(t_c)_{reciprocity} < 0,\\ &\bar{d}(t_c)_{reputation} < 0, \text{and} \\ &\bar{d}(t_c)_{mixed} < 0 \end{aligned} \tag{3}$$

to

$$\begin{aligned} &\bar{d}(t_c)_{reciprocity} < 0,\\ &\bar{d}(t_c)_{reputation} < 0, \text{with} \\ &\bar{d}(t_c)_{mixed} \geq 0. \end{aligned} \tag{4}$$

Through the evaluation of Spearman's ρ for both the reciprocity and reputation schemes, we confirm that these two schemes are monotonically decreasing, with $\rho = -1$ within the scope of our simulations. Hence, one needs only to check the condition for the change in average fitness index for the mixed scheme. In Fig. 4, we plot t_c for discrete values of population size $|N| \in (10, 1000)$, where the result is averaged over 10^3 independent simulations.

Statistical analysis of the data plotted in Fig. 4 suggests that the emergence of Parrondo's paradox given by t_c is monotonically related to the population size $|N|$, Spearman's $\rho = 1.00$. Furthermore, there is a strong linear relationship between the two variables, Pearson's $r = .99$, $p < .00001$, where the linear regression, $t_c = 11.19|N| + 10.63$ has a mean absolute percentage error (MAPE) of 0.89%.

Next, we examine how the proportion of interactions affects the outcome. In our previous simulations, we only considered stochastic switching between the reciprocity and reputation schemes with equal probability. We now introduce p, the proportion of "reputation" interactions among $t = 10^4$ independent interactions. Fig. 5 shows that pure schemes are collectively destructive for the group, while a large domain of mixed strategies can lead to favorable outcomes for $0.0951 \lesssim p \lesssim 0.7793$. Through quadratic spline interpolation, we find that the best outcome is achieved for $p \approx 0.4906$, which is close to the stochastic switching of equal proportion.

We now examine the result of Parrondo's paradox on two theoretical random networks. The results for the scale-free and small-world networks with $|N| = 100$, $T = 2000$, $K = 10^6$ are presented in the Appendix. No discernible differences are observed by changing the parameters of the small-world network. On the other hand, the scale-free network gives us the opportunity to examine how individual capital is affected by the number of neighbors each agent has and how well their connections are associated with others. In network science, these are called the degree of a node, and clustering coefficient, respectively. In the following discussion, stochastic switching of both schemes with equal probability is employed. We plot the individual capital of 10^7 agents, simulated from $K = 10^5$ experiments of $|N| = 100$ with $T = 2000$ interactions each in Fig. 6. The results show that there is a positive correlation between the degree and the fitness index of the agent. This is analogous to Matthew's effect, where the well-connected stands to gain the most. Furthermore, Fig. 7 shows that agents who themselves are connected to other agents who on average are connected to agents who are weakly clustered, tend to do better than those being part of an isolated or densely clustered group. This provides evidence supporting the theory of structural holes from sociology, introduced by Burt [34]. Burt's theory posits that an individual who acts as a bridge between two or more distinct, yet closely connected groups of people could gain important comparative advantages.

From these results, we thus assert that the collective outcome is dependent only on the social rules imposed on the network. In contrast, the individual outcome depends on the number of connections of the agents, who are bridges to distinctly connected groups.

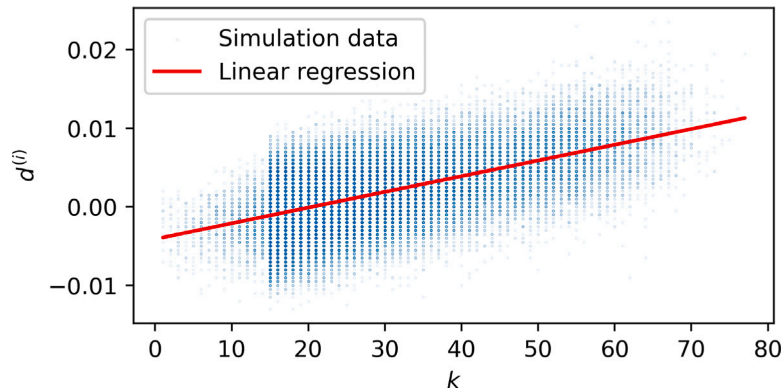

Fig. 6. The fitness index of individual agents $d^{(i)}$ as a function of the degree of the agent k in the mixed scheme. A weakly positive correlation is observed, where agents with higher degrees can expect higher fitness.

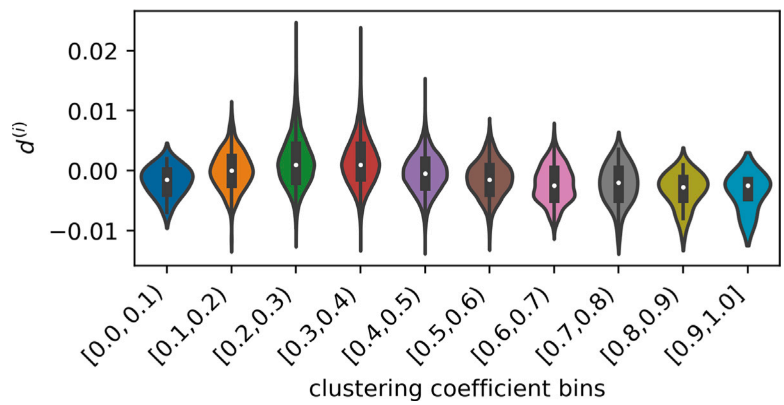

Fig. 7. The fitness index of individual agents as a binned function of the clustering coefficient of the agent in the mixed scheme. The domain with the highest average fitness index is from the bins $[0.2, 0.3)$ and $[0.3, 0.4)$.

4. Conclusion

In conclusion, while Parrondo's paradox may be counter-intuitive, it may shed light on certain sociodynamical phenomena through careful study of the underlying dynamics of these nonlinear systems. This article revealed that reciprocity or reputation alone may be a suboptimal strategy in social settings, but combining strategies may produce beneficial outcomes. This article sets the stage for extending our findings to other social networks and evolutionary game theory. Future research can also incorporate statistical mechanics to explore the physics of social dynamics through various reciprocity and reputation strategies. Researchers can have a predictive edge by building a model based on Parrondo's paradox that successfully predicts the behavior of agents. This predictive capability can guide experimental setups, helping experimental groups design their investigations more effectively. By defining conditions where Parrondo's effect is maximized, the model can also help in the effective validation of social theories. We have introduced a pioneering framework that elucidates the complex interrelations between reciprocity and reputation across network structures, offering a versatile tool for probing a wide array of novel scenarios. Within this framework, we have demonstrated a sociodynamic application of Parrondo's paradox–a counterintuitive phenomenon whereby combining two losing strategies can result in a winning outcome–specifically in the realms of reciprocity and reputation. This article has unveiled an important mechanism: how stochastic shifts in individual social behaviors can paradoxically coalesce into collective advantages. This revelation underscores the enigmatic yet profound influence of specific social structures and rules on group welfare. In particular, Parrondo's paradox emerges within a Goldilocks domain described by t_c, and it is worth noting that there is a strong linear relationship between this critical value and the population size. Our findings underscore that while social rules governing the interactions within a network dictate the collective outcome, individual outcomes hinge on the connectivity of agents serving as bridges between uniquely linked groups.

CRediT authorship contribution statement

Joel Weijia Lai: Conceptualization, Formal analysis, Writing – original draft, Writing – review & editing. **Kang Hao Cheong:** Conceptualization, Formal analysis, Supervision, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.chaos.2023.114386.

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