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ABSTRACT

Individuals can make choices for themselves that are beneficial or detrimental to the entire group. Consider two losing choices that some individuals have to make on behalf of the group. Is it possible that the losing choices combine to give a winning outcome? We show that it is possible through a variant of Parrondo's paradox—the preference aggregation Parrondo's paradox (PAPP). This new variant of Parrondo's paradox makes use of an aggregate rule that combines with a decision-making heuristic that can be applied to individuals or parts of the social group. The aim of this work is to discuss this PAPP framework and exemplify it on a social network. This work enhances existing research by constructing a feedback loop that allows individuals in the social network to adapt its behavior according to the outcome of the Parrondo's games played.

One of the aims of studying social dynamics is to help improve group performance, communication, and consensus. The use of computational modeling to examine sociodynamical phenomena is a topic of immense interest. Computational models of social structures often vastly simplifies the aforementioned complexity that is present in social dynamics. However, the benefits of computational modeling eliminates the need for real-time experiments and unwanted influence from external factors. The modeling of interaction between multiple agents, connected through some form of social construct or mode of communication, can be modeled through network science. Network science is useful for investigating real-world interactions because it is able to emulate most mathematical models of social organizations with the needed complexity. As an added advantage, factors such as influence, resolution, and cooperation can be incorporated, demonstrating the robustness of using network models. In this paper, we investigate if it is possible for two losing decisions to combine to improve welfare of a social group? This is an open problem that has been at the core of studying Parrondo's paradox and social dynamics.

I. INTRODUCTION

The use of complex computational modeling to examine sociodynamical processes is a topic of immense interest.[1–3] Coupled by network science, these techniques allow us to investigate real-world interactions because they are able to provide the desired complexity in most mathematical models of social organizations.[4] Two losing games can be combined in a certain manner to give a winning outcome. This is known as Parrondo's paradox.[5–7] While Parrondo's paradox has traditionally been applied to various aspects of science and engineering,[8–12] quantum systems,[13–17] epidemiology,[18] and biology,[19–25] the cooperative variant of Parrondo's paradox has been used to investigate sociodynamical systems. A comprehensive narrative review on the existing literature can be found in a recent work.[26] In important social interactions such as voting,[27–30] it is observed that under a democratic form of choosing which losing game to play, it results in a diminishing outcome with increasing number of players. If the ensemble decides according to the choice of a dictator, it brings benefit to the group. Concepts from Parrondo's paradox can also be found in the matching problem, where it is possible to benefit despite being one of the weakest in the group.[31–33] The cooperative variant of Parrondo's paradox has also shed light on the potential of redistribution of resources,[34] and in systems engineering, a macrocanonical way to alleviation of network congestion.[35]

A common way of describing Parrondo's paradox is through the equation game A + game B = Outcome. Here, games A and B are losing games; however, under some form of switching, represented by +, the outcome is a winning game. In the past, research on Parrondo's paradox applied to networks has considered changes to either the Parrondo's games or Parrondo's switching. For example, changing the games to cooperation and competition behaviors[36–39] or changing the switching based on the network agent's homogeneous behaviors involving the use of voting.[28] However, in these studies, the agents in the network have homogeneous behaviors. This means that agents are "forced" to follow the same set of social rules provided by Parrondo's games and switching without means of adapting to the outcome of previously played games. Thus, the behavior of each agent does not evolve with the number of games played. Furthermore, each of these games is independently designed with no coordinating framework to guide subsequent research on how to further develop their work. To close these gaps, this paper aims to design a framework that considers the preference of each agent in the network, the options given, the decision arrived from making a choice, and a feedback loop that allows agents to change behavior. This framework will serve as an important structure for inspiring future research related to Parrondo's games in the network setting.

We first discuss the capital-dependent Parrondo's paradox (see Fig. 1) played by a single player with an initial capital of C_0. Given that A and B are two losing games. In game A, the player throws a biased coin A and gains (or loses) one unit of capital according to the outcome of the biased coin. In the case of Parrondo's games, the probability of winning game A is p. For the second game, game B, the player tosses a biased coin $B1$ if the player's capital is a multiple of three; otherwise, biased coin $B2$ is tossed. The probability of winning with coin $B1$ is p_1 and with coin $B2$ is p_2. Similarly, the player wins (or loses) 1 unit of capital depending on the outcome of the coin toss. It can be shown that for the parameter space $p = 1/2 - \epsilon$ and $0 < \epsilon \ll 0.5$, game A is, in the long run, a losing game because the expected capital of the player will decrease. For $p_1 = 1/10 - \epsilon$ and $p_2 = 3/4 - \epsilon$ with the same ϵ domain, game B is proven to also be a losing game.[5]

When played randomly, the player has no information about one's capital and chooses to play game A or B with equal probability at each discrete time t, both losing games combine, under random switching, to give a winning outcome. This is Parrondo's paradox. The proof is provided in Appendix A.

The seminal capital-dependent Parrondo's game is designed to be played by an individual. If the player has full knowledge of one's capital, the player can always make an informed choice by choosing the game that has the lowest risk of losing at discrete time t. That is, according to the game rules, an individual will prefer game A if one's capital is not divisible by 3 since the player is forced to

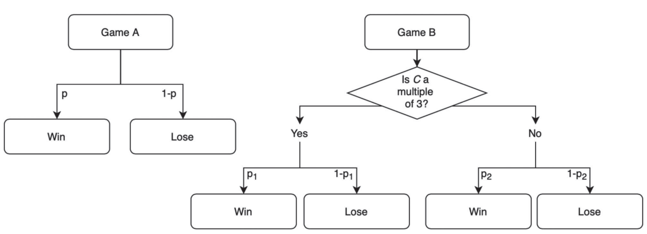

FIG. 1. Capital-dependent Parrondo's game.

toss the "bad" coin if the player chooses game B, which has a higher probability of resulting in the loss of 1 unit of capital compared to when playing game A. By the same logical reasoning, an individual will prefer game B if one's capital is divisible by 3 because there is a higher potential to lose when game A is chosen. Thus, in the case of an individual playing Parrondo's games, it is possible, with knowledge and information about one's capital at any point in time, to design a strategy that results in the gain of capital.

However, consider the case with multiple players, each with individual starting capitals. When more individuals are involved, it is not always possible to know the complete set of information (i.e., the capital of all players). In such a case, we only need to consider the information available to a subset of the group. Further restrictions (or rules) can be added in groups, such as requiring a subset of players to play the same game at each time step. In such complex circumstances, the gain in net capital across the entire group may not be clear, and analytical derivations as proofs may not be possible as well, leading to the need for computational simulations. As mentioned, increased social construct complicates the game and decision-making process. Furthermore, the game's outcome can be affected by how and who makes the choice, as well as the rules that the group follows. Thus, to exemplify and investigate this, we need a robust way to model social groups.

One approach is through scale-free networks. Scale-free networks[40] are rare in nature but are commonly regarded as essential for studying many areas of science and engineering, such as the topology of web pages and links, the collaboration network of Hollywood actors, and the peer-reviewed scientific literature. In particular, social groups and online social networks do bear resemblance to scale-free networks. A scale-free network can be constructed by progressively adding nodes to an existing network and introducing links to existing nodes with a preferential attachment so that the probability, P, of linking to a given node i is proportional to the number of existing links that node has.[41]

While there are multiple ways to generate an undirected scale-free network, we limit our discussion to one type of network generating algorithm. In particular, we will employ the use of the Barabási–Albert random scale-free network (BA model)[42] as the main network structure in our study of preference aggregate Parrondo's paradox for undirected social networks. In the BA model, new nodes join the network by attaching m (undirected) edges to other nodes, according to a linear preferential attachment. This means that the probability $P(k)$ that a link of a new node connects to node i depends on the degree k_i,

$$P(k_i) = \frac{k_i}{\sum_j k_j}, \tag{1}$$

where k_j is the degree of all existing nodes.

There are other social structures better encapsulated by directed scale-free networks. These include networks with some form of social influence such as in a hierarchical society or on social platforms like the Twitter network. We will employ the use of a directed scale-free network[43] in our investigation.

In this paper, we want to investigate how different rules and feedback processes can affect the outcome of playing Parrondo's games in a social network. The paper is organized as follows: in

Sec. II, we present the preference aggregate model and introduce three experiments that follow the model. We then present the counter-intuitive results in Sec. III for the various experiments and provide explanations for these results. Finally, we conclude and provide insights into extending the work in Sec. IV.

II. PREFERENCE AGGREGATE MODEL

Let $N = \{1, 2, \ldots, n\}$ represent a set of individuals in a group deciding which Parrondo's games to play. Each individual $i \in N$ is a node in the network and is connected to a subset of individuals $M_i \subseteq N\backslash\{i\}$ by either in-directed or out-directed edges for a directed network or edges for an undirected network. The preference aggregate model comprises three distinct steps for each iteration:

1. Aggregate rule. A list of preferences across individuals $(p^{(1)}, p^{(2)}, \ldots, p^{(n)})$, where $p^{(i)} = \{-1, 1\}, \forall i$, is called the preference profile. At discrete time t, each individual has a preference of which of the Parrondo's games to play, denoted by $p^{(i)}(t)$, where $p^{(i)} = 1$ represents a preference for game A and $p^{(i)} = -1$, a preference for game B. While an individual can abstain from a preference, this option is not considered for simplicity (furthermore, it is not applicable for Parrondo's games as players must make a decision to play either game). The *aggregate rule* is a social rule determined by a mathematical function that takes the preference profile of a subset of individuals in a social network and assigns an option, i.e., $f: \mathbb{R}^m \to \mathbb{R}$, where $m \leq n$. The function f outputs an *option*.
2. Decision rule. The option is then passed to the *decision rule*. The decision rule behaves like a choice function that either accepts or rejects the option by assigning a decision. The choice function returns a boolean output, i.e., $g : \mathbb{R} \to \{-1, 1\}$. If $g = 1$, then the decision is to play game A, otherwise, the decision is to play game B. This decision can be applied to any subset of the group. In the case where it is applied to a group of individuals, we call this a bloc decision.
3. Feedback loop. The *feedback* is a set of rules that is applied to a subset of the network and modifies the network in preparation for the next iteration.

Each of these steps can be separately designed to model different social situations. Any of the above steps can form the bulk of the computational memory and take the majority of computation time. In most real-world scenarios, the three steps are interconnected, which gives real-world sociodynamical situations the high complexity, as discussed previously. In Subsections II A–II C, we will introduce the variations of the three steps and how preference aggregate Parrondo's paradox can utilize the novel preference aggregate model.

Assigned to each individual $i \in N$ is the confidence index $\mu^{(i)}(t)$, representing the self-confidence that i has over one's own preference, satisfying $\mu^{(i)}(t) \in [0, 1], \forall i$. The confidence index may evolve with discrete time t. If individuals $i, j \in N$ are connected, the edge $j \to i$ represents j having an influence over i. To each directed edge, it is assigned the weight $\psi_{ji}(t)$. ψ_{ji} is the influence index representing the influence that j has over i's option, satisfying $\psi_{ji} \in [0, 1]$, $\forall i \neq j, j \in \Delta^{-}(i)$, where $\Delta^{-}(i)$ is the set of nodal successors of node i. In the case where the network is undirected, then for every

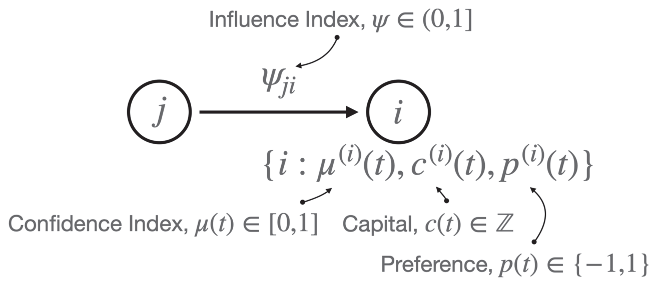

FIG. 2. Schematic describing the parameters assigned to nodes and edges of the directed scale-free network.

edge $j \to i$, there exists an edge $i \to j$ for which $\psi_{ji} = \psi_{ij} = 1$. Furthermore, each individual i starts with an initial capital $c^{(i)}(0)$, with $c^{(i)}(t) \in \mathbb{Z}, \forall t$, denoting the capital as a function of time. Hence, the initial average capital per individual of the social group C_0 is given by

$$C_0 = \frac{1}{|N|} \sum_{i=1}^{n} c^{(i)}(0). \tag{2}$$

We further define $\langle C(t) \rangle$ as the expected capital per individual at discrete time t and $\langle C \rangle$ as the expected capital per individual after $t = 100$. Both these quantities are averaged over 10^6 simulations if unspecified. An illustrative representation of the setup for a directed network can be found in Fig. 2.

To illustrate how the model can be used to investigate social dynamics when playing Parrondo's games, we will now introduce three experiments. The first two are individual games played on an undirected network. This means that at each time step, only a single individual plays the games as described in Sec. I. By modifying the aggregate rule and feedback loop, we investigate how such modifications affect $\langle C \rangle$. The third experiment is designed to comprise of bloc games played on a directed network. We will use the third experiment to explore complex rules and feedback loops.

A. Experiment 1: Ill-informed advising the ill-informed

The first application of aggregate choice is "ill-informed advising the ill-informed." Here, an "ill-informed" individual is described as an individual who has no knowledge of one's capital or the outcome of the coin toss Parrondo's games. As such, the "ill-informed" individual always selects a random game. The "ill-informed" aggregates the choice of other "ill-informed" (who individually have no knowledge of their capital or the game that benefits them). After playing that game, the player remains clueless and continues being an "ill-informed" advisor to one's neighbors. To simulate this, we consider an undirected network.

The **aggregate rule** for each iteration in this experiment is given by

$$f(p^{(i)}, p^{(M_i)}) = \frac{1}{|M_i| + 1} \left[\alpha p^{(i)} + (1 - \alpha) \sum_{\forall j \in M_i} p^{(j)} \right], \tag{3}$$

where $j \in M_i \subseteq N\backslash\{i\}$ are nodes connected to node i, $|M_i|$ is the cardinality of set M_i, the size of i's connections, and α is a parameter liken to the weight that i assigns to the preference profile of its neighbors. If $\alpha = 1$, then individual i only considers one's own preference; otherwise, on the other extreme, if $\alpha = 0$, then individual i only considers the collective preference of one's connections. The aggregate rule outputs an option, which is passed on to and decided upon by the decision rule.

The **decision rule** here is simplified to simulate the naïve decision-making process of "ill-informed" individuals. Specifically,

$$g = \begin{cases} 1 & \text{if } f \geq 0, \text{ i.e., } i \text{ decides to play game A,} \\ -1 & \text{if } f < 0, \text{ i.e., } i \text{ decides to play game B.} \end{cases} \tag{4}$$

Finally, for the **feedback loop**, after individual i plays the game as stipulated by the decision rule, $c^{(i)}(t+1)$ is updated accordingly, and $p^{(i)}(t+1)$ will be randomized with equal probability of playing either game A or B. We will vary α to investigate $\langle C \rangle$.

B. Experiment 2: "Cautious" vs "risk-taking" vs "reckless"

In the second experiment, we simulate three well-known social decision-making adages, "once bitten, twice shy," "risk-taking," and "reckless." The saying "once bitten, twice shy" describes a bad past event that leads to caution. One might consider that in a game of chance, as described by Parrondo's games, a strategy of caution will not deviate much from a strategy of taking risks or being reckless because the outcomes of Parrondo's games are randomized. However, we show that being reckless benefits the group more than taking risk and practicing caution bring the most detriment to the group. The **aggregate and decision rules** are similar to the previous experiment. For the **feedback**, individual i modifies its preference according to the result of the game selected by the decision rule. First, for "cautious," the feedback loop is given by

$$p^{(i)}(t+1) = \begin{cases} -p^{(i)}(t) & \text{if } \Delta c^{(i)} < 0 \text{ and } p^{(i)}(t) = g, \\ p^{(i)}(t) & \text{if} \Delta c^{(i)} < 0 \text{ and } p^{(i)}(t) = -g, \\ \text{rand}\{-1, 1\} & \text{otherwise.} \end{cases} \tag{5}$$

$\Delta c^{(i)}$ denotes the change in capital at time step t and rand$\{\cdot\}$ denotes a random choice between the options with equal probability. For the "risk-taking," the feedback loop is given by

$$p^{(i)}(t+1) = \begin{cases} p^{(i)}(t) & \text{if } \Delta c^{(i)} < 0 \text{ and } p^{(i)}(t) = g, \\ p^{(i)}(t) & \text{if } \Delta c^{(i)} > 0 \text{ and } p^{(i)}(t) = -g, \\ \text{rand}\{-1, 1\} & \text{otherwise.} \end{cases} \tag{6}$$

Last, to simulate the "reckless," the feedback loop is given by

$$p^{(i)}(t+1) = \begin{cases} p^{(i)}(t) & \text{if } \Delta c^{(i)} < 0 \text{ and } p^{(i)}(t) = g, \\ p^{(i)}(t) & \text{if } \Delta c^{(i)} > 0 \text{ and } p^{(i)}(t) = -g, \\ -p^{(i)}(t) & \text{if } \Delta c^{(i)} < 0 \text{ and } p^{(i)}(t) = -g, \\ -p^{(i)}(t) & \text{if } \Delta c^{(i)} > 0 \text{ and } p^{(i)}(t) = g. \end{cases} \tag{7}$$

The first two scenarios, for the "reckless" individual, are similar to the feedback from the "risk-taking" individual. However, the "reckless" will risk it further by considering choices that are not favorable. Explanation to the setup and real-life decision-making heuristic can be found in our prior work.[44]

Similarly, for this experiment, we vary α to investigate $\langle C \rangle$. The difference between experiments 1 and 2 is thus in the feedback loop. While the individuals in experiment 1 update their preferences randomly, individuals in experiment 2 update their preferences based on their social behavior—either cautious, risk-taking, or reckless.

C. Experiment 3: Weighted influence and confidence

Thus far, we have not considered the confidence index $\mu^{(i)}$. In reality, individuals make decisions based on the confidence that they have on their own preference. Unlike experiment 1 and 2, individuals give weight to other individuals' influence. For example, a supervisor's preference might influence someone more than a fellow colleague. In this experiment, we introduce a possible method of accounting for these real-world scenarios by considering a directed, weighted network.

The **aggregate rule**, at discrete time t, for this experiment is given by

$$f(p^{(i)}, p^{(M_i)}) = \begin{cases} p^{(i)} & \text{if } i \text{ have no predecessors,} \\ \text{rand}\{-1, 1\} & \text{if } i \text{ have no predecessors and } \mu^{(i)} < \eta, \\ \frac{1}{|M_i|+1}\left[\mu^{(i)} p^{(i)} + \sum_{\forall j \in M_i} \beta^{(j)} p^{(j)}\right] & \text{if } i \text{ have predecessors,} \end{cases} \tag{8}$$

where $j \in M_i \subseteq N \backslash \{i\}$ the predecessors of node i, $|M_i|$ is the cardinality of set M_i, and β is the weighted preference of i's predecessors satisfying

$$\beta^{(j)} = \frac{\psi_{ji}}{\sum_{j'=1}^{|M|} \psi_{j'i}} - \frac{\mu^{(i)}}{|M|}. \tag{9}$$

This aggregate rule considers the weight of influence that each predecessor of i has on i's option. The first two cases for Eq. (8) cover the case for an individual who has no predecessors. If the confidence index is high, then i's option will be the same as its preference; otherwise i will randomly choose an option. As a way of illustration, we have let $\eta = 0.75$ in our simulations.

The **decision rule** is modified to take into consideration i's own influence one's successors. If individual i does not have any successors, then only i plays the game according to the decision rule in Eq. (4). If individual i has successors k, then the successors will play the same game as i if $\psi_{ik} > \gamma$, otherwise, successor k will not play any games this round. γ is a parameter that we can vary. If $\gamma = 0$, then all of i's successors will play the same game as i for that round. If $\gamma = 1$, then only individual i plays the game decided by the decision rule for that round.

Last, for the **feedback loop**, the confidence of individual i improves if i gained capital in that round, and decreases when the converse happens. The confidence index is modeled as such

$$\mu^{(i)}(t+1) = \begin{cases} \tanh\left(\frac{5\mu^{(i)}(t)}{2}\right) & \text{if } i \text{ gained capital,} \\ 0.1\mu^{(i)}(t) & \text{otherwise.} \end{cases} \quad (10)$$

Here, the penalty on the confidence is significant as a loss in capital drastically decreases the confidence one has on one's own preference; while a gain in capital eventually saturates one's confidence. Finally, we examine how various values of γ in the decision rule affect $\langle C \rangle$ if $p^{(i)}$ at the next iteration is set to be randomized or the last game that resulted in capital gain for i.

III. RESULTS

We employ `NetworkX 2.5`[45] package available within the Python language for the setup. For consistency and comparison, we set the bias of Parrondo's games to be $\epsilon = 0.01$ throughout all experiments, unless explicitly mentioned. The BA network is generated with the following properties: Node size (number of players) $|N| = 500$ with an average degree $\langle k \rangle = 3.984$, this is achieved by setting the built-in random graph function with the following inputs: `barabasi_albert_graph(500,2)`. These parameters also fit the results observed for domain level interactions on the Internet and social networks.[46,47] For each network, set each individual node's initial capital $c^{(i)}(0) \in \mathbb{Z}$, drawn uniformly from the interval $[-\$, \$]$. Without loss of generality, we choose to set $\$ = 5$ for our simulations as a way of illustration. In general, we are only interested in the change of capital with time t.

The same can be carried out for the directed scale-free network `scale_free_graph(500)`, with the parameters set as the default. Specifically, the probability for adding a new node connected to an existing node chosen randomly according to the in-degree and out-degree distributions is 0.41 and 0.05, respectively. The probability for adding an edge between two existing nodes is 0.54. An existing node is chosen randomly according to the in-degree distribution and another is chosen randomly according to the out-degree distribution. Finally, the bias for choosing nodes from in-degree distribution is 0.2, with no bias set for choosing nodes from out-degree distribution. The other variables $\mu(t)$, $p(t)$ and ψ_{ji} are assigned random values drawn uniformly from their respective domains.

To obtain the results reported in Figs. 3 and 5, we generated the BA and directed scale-free networks, respectively and ran 10^6 game simulations per discrete time step. Each game simulation consists of a complete run of a preference aggregation iteration comprising of the aggregation rule, decision rule and feedback loop. To obtain the results reported in Fig. 4, the experiment was modified to allow α to vary, because of computational time, we took 10^5 simulations instead.

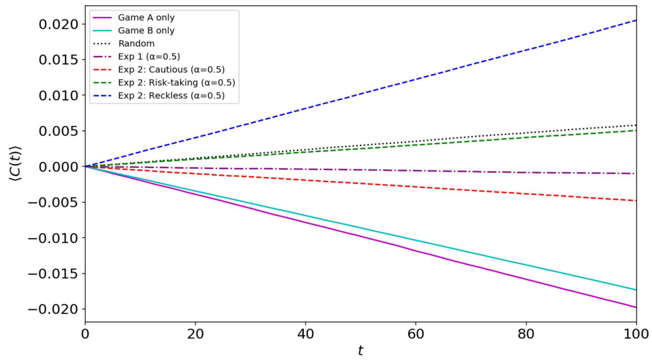

FIG. 3. Expected capital per player $\langle C(t) \rangle$ against time for a BA network of size $|N| = 500$. Note that both games A and B are losing (decreasing with time), so is the case for Experiment 1 and Experiment 2 (Cautious) for $\alpha = 0.5$. For the two cases from Experiment 2, i.e., "Risk-taking" and "Reckless," the Parrondo's effect is observed for which two losing games combine to give a winning outcome.

A. Experiments 1 and 2

In both experiments 1 and 2, only the randomly selected individuals play the preference aggregation Parrondo's games at each time step. The result of the simulations are presented in Fig. 3 for $\alpha = 0.5$ in all cases. The significance of $\alpha = 0.5$ denotes an equal share between i's own preference and the cumulative preference of all of i's connections. We have examined how α affects $\langle C \rangle$ in Fig. 4.

We make two observations from Figs. 3 and 4.

- *Observation 1*: $\langle C \rangle$ for the network of "Ill-informed" individuals is at best equivalent to the random strategy as $\alpha \to 1$.
- *Observation 2*: On the spectrum of α, "Risk-taking" and "Reckless" feedback loops yield better outcome than "Cautious" feedback loop.

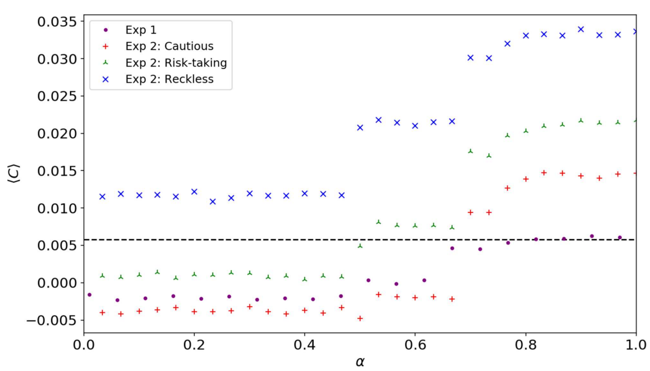

FIG. 4. Expected capital per player, $\langle C \rangle$, against α for a BA network of size $|N| = 500$. The capital per player plotted here is for $t = 100$ and averaged over 10^5 simulations each. The black dashed line represents the gain from the random switching strategy.

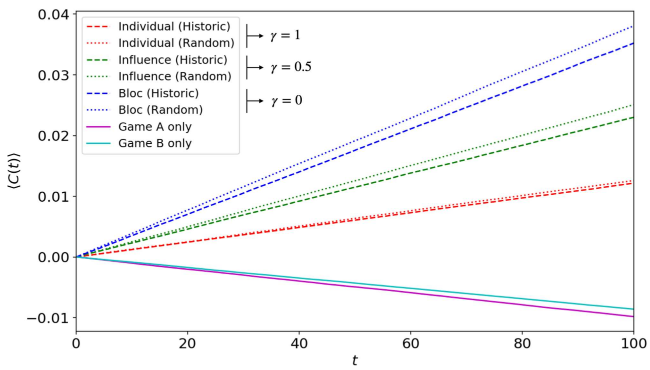

FIG. 5. Expected capital per player $\langle C(t)\rangle$ against time for a scale-free directed network of size $|N| = 500$. Note that both games A and B are losing (decreasing with time). For all cases, the Parrondo's effect is observed for which two losing games combine to give a winning outcome. The labels "Individual," "Influence," and "Bloc" are description based on the value of γ. The labels "Historic" and "Random" describe the feedback loop in which the randomly selected individual i decides its new preference.

B. Experiment 3

Experiment 3 uses the same preference aggregate framework but applies a different aggregate and decision rule and explores a feedback loop for the confidence index. In this experiment, we vary γ, the threshold value for which a randomly selected individual i will exert its influence on its successors to play the same game at each discrete time. Thus, if $\gamma = 0$, then we have bloc application of preference aggregation Parrondo's paradox (PAPP), where all of i's successors plays the same game as i (we denote this case as "Bloc"). On the other extreme, if $\gamma = 1$, then i does not exert its influence on its successors, hence, only the randomly selected individual i plays the game (we denote this case as "Individual"). We also simulated the case for $\gamma = 0.5$, denoting this case as "Influence." The randomly selected individual i also updates its preference based on either a random selection ("Random") or the last game played ("Historic"). The results of these simulations are presented in Fig. 5.

We make two more observations from Fig. 5 and Table I.

- *Observation 3*: Bloc applied PAPP ($\gamma = 0$) outperforms PAPP for the individual ($\gamma = 1$). That is, $\langle C\rangle$ decreases linearly with γ.
- *Observation 4*: For all three cases in experiment 3, despite playing an estimated equal proportion of game A to game B, a change in the decision rule subjected to the same aggregate rule and feedback loop allows one form of PAPP to outperform another.

C. Discussion

Observation 1 describes the case of "ill-informed advising the ill-informed." As $\alpha \to 1$, the aggregate rule tends toward a "selfish" decision-making process as more weight is given to individual preference. This is equivalent to the scenario described in the random choice game, thus the results for "Ill-informed" is at best the random choice game. As it turns out, in a network where ill-informed individuals are providing feedback to other ill-informed individuals, the possible gain is small but achievable. The use of the aggregate rule to "Ill-informed" is analogous to the lack of information provided to individuals in a network, in our case, the agents in the network are "ill-informed" about the significance of their choice to their capital, which can be paralleled to individual welfare. Our results reflect that in the case of "ill-informed advising the ill-informed," gains can only be made if the weight of the decision is shifted toward the individual. This is akin to shutting out social chatter (influence) from other ill-informed individuals and making a naïve choice at random. Collectively, this is beneficial for the group as it can turn losses into gains.

On *Observation 2*. We observe that α, how an individual places weight on the preferences, does indeed affect the outcome of playing Parrondo's games. Recall that the α-spectrum provides information on how individuals weigh their preferences vis-à-vis its connections. For small α, the individual places more weight on its connections' preferences, allowing it to majorly influence its decision. For large α, the individual is now more keen to play the game preferred rather than placing weight on its connections' preferences.

First, we comment on the network of cautious individuals. In this network, individuals adopt the "once bitten, twice shy" strategy by reacting only when losing capital. By adopting a cautious behavior, the individuals in this network can lessen their losses by aggregating preferences, thus, not losing as much as compared to playing games A or B individually. However, it is only when individuals place a significant weight on their preference, $\alpha \gtrsim 0.7$ that we observe the breakthrough required to turn a net gain in capital. A small gain in capital is expected because the individuals are simply playing based on past information. Past information is not useful in predicting future outcomes in randomized coin toss games like Parrondo's games.

Next, the trend observed for the network of cautious individuals is unlike the network of risk-taking or reckless groups that employ the use of paradoxical switching strategies to cause two losing choices to combine to give a winning outcome. We observe that both the network of risk-taking individuals and reckless individuals always outperform the network of cautious individuals. Contrary to basic intuition, this is an unexpected result as one will expect risk-taking and reckless individuals to lose out. However,

TABLE I. Summary table of the expected capital per player at $t = 100$, $\langle C\rangle$ and the proportion of game A played during the simulation of each case in experiment 3. The total number of simulation per discrete time is 10^6.

	Individual (historic)	Individual (random)	Influence (historic)	Influence (random)	Bloc (historic)	Bloc (random)
$\langle C\rangle$	0.012 16	0.012 60	0.023 00	0.025 11	0.035 24	0.038 08
Prop. of game A	49.99%	50.00%	49.83%	50.00%	49.74%	50.02%

TABLE II. Summary table of the expected capital per player at $t = 100$, $\langle C \rangle$ and the proportion of game A played during the simulation of experiments 1 and 2. The total number of simulations per discrete time is 10^6 and $\alpha = 0.5$.

	Expt. 1 "Ill-informed"	Expt. 2 "Cautious"	Expt. 2 "Risk-taking"	Expt. 2 "Reckless"
$\langle C \rangle$	-9.8560×10^{-4}	-4.8103×10^{-3}	5.0474×10^{-3}	0.020 513
Prop. of game A	44.66%	46.13%	46.62%	47.90%

because Parrondo's games are fundamentally probabilistic games and counter-intuitive, using rational feedback does not facilitate in improving capital. We observe that the most irrational feedback taken by reckless individuals indeed leads to the overall best outcome. Comparing the risk-taking and reckless network of individuals, taking a bold strategy of consistently switching preferences, the reckless individual may be switching away from an unfavorable game more often than the individual who takes risks and sticks to the same unfavorable game. Concepts of risk-reward phenomena can be used in game and mechanism design[48] and have relevance in game theory, especially in the win-stay lose-shift strategy commonly adopted in discussing the prisoner's dilemma.[49]

From the results of experiments 1 and 2, it is clear that if an individual decides by placing greater weight on its connections, the noise from its connections leads to lesser gains than deciding based on individual preferences. This suggests that capital-dependent Parrondo's games with partial information, since the actual capital does not affect the preference of games, are best played individually. The jump in the average capital seen in Fig. 4 suggests that there is a phase change taking place as α is varied. The critical values of α are $\alpha = 0.5$, and $\alpha = 0.7$. These phase changes are oddly familiar as seen in thermodynamics.

On *Observation 3*. As understood from capital-dependent Parrondo's paradox, the ratcheting combination of games A and B results in two losing games leading to a winning outcome. However, there is indeed a preferred game if full information is made known to all the individuals of the network. The design of experiment 3 is such that depending on the parameter γ, the successor k of individual i is forced to play the same game as i if $\psi_{ik} > \gamma$. The act of "forcing" successors to play the game can switch the unfavorable preference of k for a favorable preference, i.e., if the capital $c^{(k)}(t) \mod 3 \equiv 0$, then the favorable preference is game B as the probability of winning is higher. The act of "forcing" can equally switch a favorable game for an unfavorable game. As this happens in equal proportion, the net effect is thus only dependent on the number of individuals influenced by i. That is to say, the more individuals playing Parrondo's game at each discrete time, the higher the capital. Furthermore, since ψ_{ik} is random and uniformly drawn from the interval $[0, 1]$ the relationship between γ and the number of individuals who are subjected to play the game is linear. Hence, we conclude that $\langle C \rangle$ decreases linearly with γ.

Observation 4 is critical in ensuring that our simulations and experiments remain in the realm of Parrondo's games. The beauty of Parrondo's paradox is that the random selection, not necessarily with equal probability, of two losing games, can result in a winning outcome. A further observation of the results in Tables I and II also show that there is no discernible relationship between the proportion of game A played and the expected capital per player $\langle C \rangle$. The fact that the extensive variety of PAPP can lead to differing outcomes despite sharing some social rules (within each experiment) makes our framework an applicable and rich extension of Parrondo's paradox that may potentially open up a new field involving sociophysics. For example, how does the average clustering of the network affect the dynamics of $\langle C \rangle$? What happens if individuals in the network have knowledge of their own capital or the capital of a subset of their connections? Having complete information can be useful to improve the performance of a complex system, but complete information is not always available or viable. Thus, works in this field involving the study of computational models to explain social phenomena is and will become increasingly useful in our highly connected and complex society. The current framework lays the groundwork for further research into investigating social dynamics of Parrondo-like decision-making processes.

This work has the potential to inspire research utilizing computational science for investigating social phenomena, especially the case of achieving gains from two losing choices. We caution that the experiments performed in this work, while based on real-world behavior, still have room for improvement. As evidence for social phenomena advances, coupled by the fusion of scientific theories with social theories, we expect this field to play a larger role across various scientific disciplines. In particular, further applications of PAPP are certainly worth exploring, especially in the case of time-varying phenomena in decision-making through dynamics networks, which more closely models real-world behavior.

IV. CONCLUSION

A framework involving preference aggregation in Parrondo's games for various social scenarios is being proposed. By employing scale-free networks, rules, and feedback loops to model social interactions, we have been able to model some real-world decision-making through the framework and simulations that have been performed. In particular, we have shown that (i) "ill-informed advising the ill-informed" is detrimental to the group and in such a case, it is more beneficial for an individual of such sort to choose a game without subjecting oneself to the influence of one's connections; (ii) practicing caution through rational switching is not an advantageous strategy; (iii) recklessness can bring rewards in paradoxical games; last, (iv) bloc application is more favorable than individual application of Parrondo's games in a random network. The balance of the computational modeling and social experiments can facilitate the advancement of Parrondo-like scenarios. This work has shown that common social phenomenon can be exemplified through the use of PAPP to a social network. We have also shown that under certain social rules, as described in Sec. II, agents in a network structure can win through PAPP. We have demonstrated that our work is applicable in describing key social phenomena such as influence, risk-taking, and confidence. With increasing interest in areas of applying statistical physics to complex systems, interdisciplinary fields such as econophysics, and an emerging new area of sociophysics, our work highlights the rich applications of Parrondo's paradox in sociodynamical modeling for explaining many possible counter-intuitive outcomes from decision-making. For example, practical applications include the study of the emergence of social phenomena from partitioning a social network, which is a natural extension of experiment 2; as well as an extension of social decision-making by altering the decision rule through the inclusion of the evidence theory.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Joel Weijia Lai: Conceptualization (equal); Data curation (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Software (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Kang Hao Cheong:** Conceptualization (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available within the article.

APPENDIX A: PROOF OF CAPITAL-DEPENDENT PARRONDO'S PARADOX

For the capital-dependent Parrondo's games, we have three states, each corresponding to the states of $C \mod 3$, as seen in Fig. 6. In both Markov chains, the clockwise direction is the winning outcome of each game. It is straightforward to show that game A is indeed losing, that is,

$$1 - p > p \Rightarrow \frac{1-p}{p} > 1. \tag{A1}$$

For game B, it is losing if

$$(1-p_1)(1-p_2)^2 > p_1 p_2^2 \Rightarrow \frac{(1-p_1)(1-p_2)^2}{p_1 p_2^2} > 1. \tag{A2}$$

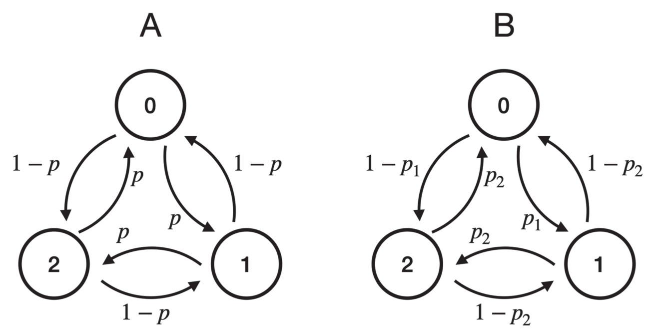

FIG. 6. Markov chain for the capital-dependent Parrondo's game.

These inequalities are indeed satisfied with the parameters chosen: $p = 1/2 - \epsilon$, $p_1 = 3/4 - \epsilon$, and $p_2 = 1/10 - \epsilon$, for $\epsilon = 0.05$; hence games A and B are indeed losing. For Parrondo's games, we can combine games A and B with some proportion of game A, given by the parameter γ. This combined game C is $C = \gamma A + (1-\gamma)B$. The stationary distribution leading to a winning game C occurs when

$$\frac{(1-q_1)(1-q_2)^2}{q_1 q_2^2} < 1, \tag{A3}$$

where $q_1 = \gamma p + (1-\gamma)p_1$ and $q_2 = \gamma p + (1-\gamma)p_2$. Thus, in the case where both games A and B are played with equal proportion, $\gamma = \frac{1}{2}$, which indeed satisfies the inequality. Thus, the capital Parrondo's game is indeed a winning game under random switching with equal proportions of games A and B played.

APPENDIX B: FRAMEWORK VIS-À-VIS PRIOR WORK

In this appendix, we show how the work presented by Wang *et al.*[36] and Parrondo *et al.*[28] can be described using the preference aggregation Parrondo's paradox framework described in the present paper.

The primary change to the work conducted by Wang *et al.* is to change the Parrondo's game. The network used is the BA network and an fully connected undirected network. In fact, our framework accepts any directed or undirected network, with weighted or unweighted edges. **Game A (competition)** involves a randomly selected principal i and a receptor j, where j is connected to i. i and j compete in a fair coin toss game. If i wins, j pays a unit of capital to i; otherwise, i pays a unit of capital to j. **Game B** is the usual capital-dependent Parrondo's game B. i decides to play game A with probability p and game B with probability $1-p$. The aggregate rule in this case is a function given by

$$f = \begin{cases} 1 & \text{if } r < p, \\ -1 & \text{otherwise,} \end{cases} \tag{B1}$$

where $r \sim U(0,1)$ is a real number drawn randomly from a uniform distribution of interval $[0,1)$. Note that in this case, the preference of principal i and receptor j is irrelevant to choosing which game i

plays. The decision rule is $g : \mathbb{R} \to \mathbb{R}$, specifically,

$$g = \begin{cases} 1 & \text{if } f = 1, \\ -1 & \text{if } f = -1. \end{cases} \tag{B2}$$

There is no feedback loop as all agents do not change behavior.

Next, we look at how our framework can also be used to encompass previous work that changes the Parrondo's switching. We consider the work performed by Parrondo *et al.*[28] In the work performed, games A and B remain unchanged, but every agent in the network has a preferred game, and the entire network has to play the same game according to a collective decision heuristic. In this case, the network N is partitioned into the decision group D and the follower group. Agents in D will aggregate their preference according to a first-past-the-post system, after which all agents in the network will play the game decided upon by the decision group. All agents in N then update their preference accordingly. The aggregate rule in this case is a function given by

$$f = \frac{1}{|D|} \sum_{i \in D} p^{(i)}, \tag{B3}$$

where $p^{(i)} = 1$ represents a preference for game A and $p^{(i)} = -1$ is a preference for game B. The decision rule applies to all agents in the network N, given by

$$g_i = \begin{cases} 1 & \text{if } f > 0 \\ -1 & \text{if } f < 0 \\ \text{rand}\{-1, 1\} & \text{otherwise} \end{cases}, \ \forall i \in N. \tag{B4}$$

The feedback loop is

$$p^{(i)}(t+1) = \begin{cases} 1 & \text{if } c_i(t) \mod 3 = 0 \\ -1 & \text{otherwise} \end{cases}, \ \forall i \in N. \tag{B5}$$

Game A is the preference for individuals whose capital is divisible by 3, as the probability of winning through game B is very small. The same can be applied for other work.

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