Chaos, Solitons and Fractals 158 (2022) 111911

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Risk-taking in social Parrondo's games can lead to Simpson's paradox

Joel Weijia Lai, Kang Hao Cheong*

Science, Mathematics and Technology Cluster, Singapore University of Technology and Design (SUTD), 8 Somapah Road, Singapore 487372, Singapore

ARTICLE INFO

Article history:
Received 26 November 2021
Revised 31 January 2022
Accepted 10 February 2022

Keywords:
Parrondo's paradox
Simpson's paradox
Social dynamics
Networks

ABSTRACT

Parrondo's paradox (inspired by the flashing Brownian ratchet) and Simpson's paradox (a statistical phenomenon) are two popular paradoxes that have attracted immense interest across many fields ranging from decision theory, evolutionary biology to social dynamics. In this article, we show that risk-taking behaviour through aggregate decision-making on Parrondo's games can lead to the emergence of Simpson's paradox. By partitioning the network of individuals according to risk-taking behaviours, we show that it is possible that the trend of capital losses from playing Parrondo's games reverses when these groups are combined—the signature of Simpson's paradox. This work reports on the emergence of the double paradox on a scale-free network and a social network, with the potential to uncover such instances in other social settings as well.

1. Introduction

Two coin toss games, each resulting in expected loss. Both games individually result in loss of capital over discrete time. However, when both games are played in combination, it results in a gain of capital. Such a phenomenon is known as Parrondo's paradox. Extensive research has gone into resolving this paradox and further research reveals underlying real-world applications across many different fields. Some examples include, in information theory where random mixing of two random sequences creates autocorrelation [1], in biological and ecological systems where species adopt various survival strategies resulting in population growth [2,3], epidemiology [4], as well as in quantum information [5–7] and quantum systems where quantum random walks has potential use for encryption [8]. Parrondo's paradox is also known to emerge when considering periodic switching between forms of the Stuart-Landau equation leading to antiresonance of unstable modes [9], in the study of chaos [10,11], and in other instances of synchronisation in network dynamics [12]. These studies, especially in its emerging application in complex systems, have thrusted Parrondo's paradox beyond simple coin-tossing games. Parrondo's paradox gave rise to an emerging field leading to diverse research of this paradoxical phenomena in network settings [13–16] and social dynamics [17]. The dynamics of such systems can be extremely complex due to their high dimensionality and inter-connectivity between individuals. At the forefront of this research is the applicability of complex computational modelling to sociodynamical phenomena [18]. The use of networks has been useful as a model as it fulfils the desired complexity required in most mathematical modelling of social structures [19].

Consider two losing games, A and B. The player starts with c_0 units of capital. In game A, the player tosses a biased coin A and wins (or loses) 1 unit of capital depending on the outcome of the biased coin. The probability of winning is $p = 1/2 - \epsilon$, where $0 < \epsilon \ll 1$; game A is a losing game because the expected capital of the player will decrease as a function of coin tosses at each discrete time t. The second game, game B, is played with two biased coins. The player tosses biased coin $B1$ if one's capital is not a multiple of three; otherwise biased coin $B2$ is tossed. The probability of winning with coin $B1$ is $p_1 = 3/4 - \epsilon$ and with coin $B2$ is $p_2 = 1/10 - \epsilon$. Coin $B1$ is a "good" coin because the probability of winning is high, for the contrasting reason, coin $B2$ is a "bad" coin. Similarly, the player wins (or loses) 1 unit of capital depending on the outcome of the coin tossed. While not immediately apparent, game B is also a losing game [20–22].

The seminal capital-dependent Parrondo's game is designed to be played randomly by an individual. However, consider the case with more players (forming a social network) are introduced to the game, each with their own starting capital. The outcome will simply scale with the number of players as the capital-dependent games are not dependent on the number of players but only on each player's capital. The complexity of the game can be modified to introduce social interaction between the players, these are imposed through restrictions or rules on the social network. In more complex situations, the gain in net capital across the entire group is not clear anymore. This enhanced social construct adds complexity to the game and the decision-making process. Moreover, the

* Corresponding author.
E-mail address: kanghao_cheong@sutd.edu.sg (K.H. Cheong).

https://doi.org/10.1016/j.chaos.2022.111911

outcome of the game changes with how and who makes the decision and the rules applied to the group, we call this Preference aggregation Parrondo's paradox [17].

As an extension of the capital-dependent Parrondo's game, we introduce the Preference aggregation Parrondo's paradox which makes use of social dynamical principles to model the playing of Parrondo's paradox on a social network. Added complexity introduced through preference aggregation leads to the possibility of the emergence of Simpson's paradox. Simpson's paradox is a phenomenon in statistics in which trends in several groups of data reverses when the groups are combined [23–25]. The phenomenon has been observed in several examples that include a study of gender bias among graduate school admissions to University of California Berkeley, a real-life medical study comparing the success rates of two treatments for kidney stones, and the batting averages of players in professional baseball. Simpson's paradox often manifests itself in social science which may skew interpretation of data and derived implications for decision-making.

We introduce a network of individuals, each with varying degree of risk-taking behaviour—"Cautious", "Risk-taker" and "Reckless" and their response to playing preference aggregation Parrondo's games will result in the emergence of Parrondo's paradox and Simpson's paradox. Risk-taking behaviour is a multidimensional facet that have been studied across a wide range of domains [26–28]. As a result, there is no single definitive agreement for how risk-taking behaviour should be modelled. However, it is generally accepted that decision-making processes taken under risk are often nonlinear and probabilistic [29,30]. For the purpose of this article, we are modelling it through a risk-taking factor β, the aggregation rule f, the decision rule g and the feedback. Considerations for nonlinear and probabilistic decision-making under risk are accounted in the construct of our model, using preference aggregation Parrondo's paradox as our framework. Next, we introduce the preference aggregation model used as a decision-making scheme for playing Parrondo's games. These schemes are modelled according to sociology and social experiments. Lastly, we show emergence of Simpson's paradox and conclude.

2. Methods

Let $N = \{1, 2, \ldots, n\}$ be a set of individuals, where $|N| \geq 2$. Each individual $i \in N$ is a node in the network and is connected to a subset of individuals $M \subseteq N \backslash \{i\}$ by edges for an undirected network. Each iteration of the preference aggregation Parrondo's games involves an aggregate rule, decision rule and feedback. At discrete time t, each individual has a game preference, $p^{(i)}(t)$ (also called the preference profile), based on Parrondo's games introduced in the previous section, where $p^{(i)} = 1$ represents a preference for game A and $p^{(i)} = -1$ represents a preference for game B. Each individual is also assigned a risk-factor $\beta \in [0, 1]$. The value of β will subsequently determine the risk-taking behaviour of the individual when playing preference aggregation Parrondo's games. A bijective, nonlinear mapping between risk-factor and behaviour is modelled through the parameter β. In addition to β indicating the risk-factor, we take the approach of having the domain of β assigned to each risk-taking behaviour being equal. Furthermore, an overlapping region is built in to allow individuals with the same β to possess different risk-taking behaviour. There are other models [30] that can be used in replacement of the one used here, but we have used a simplified mapping for illustrative purpose. In another model, Riolo et al. [31] use the tag-based mechanism where individuals will aggregate the preference only with other individuals with similar tags. The results from such a model can be found in the supplementary information. The *aggregate rule* is a social rule, determined by a mathematical function that takes the preference profile of individuals in a social network and assigns an *option*, i.e. $f : \mathbb{R}^{\nu} \to \mathbb{R}$, where $\nu \leq n \in \mathbb{N}$. The aggregation function f outputs an *option*. The *decision rule* accepts or rejects the option by assigning a *decision*. If the option is accepted, the decision function returns a boolean output, i.e. $g : \mathbb{R} \to \{-1, 1\}$. If $g = 1$, then the decision is to play game A, if $g = -1$, game B is played. Finally, the *feedback* is a final set of rules applied to a subset of the network and modifies the network in preparation for the next iteration. We will model the risk-taking behaviors using the feedback.

The aggregation rule for i at each discrete time t is

$$f(p^{(i)}) = \frac{1}{|M|+1}\left[p^{(i)} + \sum_{j\in M} p^{(j)}\right], \quad (1)$$

where $j \in M \subseteq N \backslash \{i\}$ are nodes connected to node i, $|M|$ is the cardinality of set M, the size of i's connections. Such an aggregate rule always ensures that individuals tend to place more weight on its own preference, a common phenomenon observed in risk-taking [32,33]. The aggregation and decision rules are kept similar for the three risk-taking behaviours to ensure that the only factor that affects the change in capital is purely its feedback, which itself is modelled according to the various characteristics of risk-taking behaviours.

The aggregate rule outputs an option which is decided upon by the decision rule given by several possible cases:

(i) If $p^{(i)}\left(\sum_{j\in M} p^{(j)}\right) < 0$ and $r < \exp(-1/2\beta_i)$, $r \in [0, 1]$ is a random number also known as the acceptance number, then i decides using the decision rule

$$g^{(i)} = \begin{cases} 1 & \text{if } f(p^{(i)}) \geq 0 \\ -1 & \text{if } f(p^{(i)}) < 0 \end{cases}. \quad (2)$$

This means that if individual i has a preference opposite to the sum of the preferences of its connections and i's preference is within an acceptable threshold, then i will decide to play Parrondo's games according to the option provided by the aggregate rule. This is a simplification of decision-making formerly presented in Refs. [34,35], where individuals decide on an option that is within a threshold of acceptance.

(ii) However, if $p^{(i)}\left(\sum_{j\in M} p^{(j)}\right) > 0$, then i simply chooses to play one's preference. That is

$$g^{(i)} = \begin{cases} 1 & \text{if } p^{(i)} = 1 \\ -1 & \text{if } p^{(i)} = -1 \end{cases}. \quad (3)$$

(iii) Otherwise, if the conditions in (i) and (ii) are not met, because the sum of preferences of i's connections not within the acceptable threshold, then the following decision rule is used

$$g^{(i)} = \text{rand}\{-1, 1\}, \quad (4)$$

where rand$\{\cdot\}$ denotes a random choice between the options with equal probability. In such cases, i rejects the preference aggregation and simply plays a random game. In this article, we term this as the universal decision rule.

We now model the feedback which varies according to β. Individuals $i \in N$ with $\beta \in [0, 10/24]$ are set as *"Cautious"* individuals. Cautious individuals adopt the aggregation rule given by Eq. (1) and apply the universal decision rule. Finally, using the principle of "once bitten, twice shy", which describes the action of avoiding bad situations, the feedback is

$$p^{(i)}(t+1) = \begin{cases} -p^{(i)}(t) & \text{if } \Delta c^{(i)} < 0 \text{ and } p^{(i)}(t) = g, \\ p^{(i)}(t) & \text{if } \Delta c^{(i)} < 0 \text{ and } p^{(i)}(t) = -g, \\ \text{rand}\{-1, 1\} & \text{otherwise.} \end{cases} \quad (5)$$

$\Delta c^{(i)}$ denotes the change in capital at time step t. In other words, if i loses capital by playing the game preferred (and decided upon using the decision rule), i is "bitten" and thus prefers to deviate from that game at the next iteration. Otherwise, if i loses capital from playing the game according to the aggregate rule, but is not i's preference, then i will not deviate. For the other cases, i gains capital, and will continue to randomise its preference. In short, the "cautious" individual only reacts when capital is lost, by either switching or reinforcing its preference.

Individuals $i \in N$ with $\beta \in [7/24, 17/24]$ are set as *"Risk-takers"*. The overlap of β is purposely designed to model the possibility that individuals with similar thresholds can still adopt fuzzy risk-taking behaviours. The aggregation rule for i is given by Eq. (1), adopts the same universal decision rule and has the following feedback

$$p^{(i)}(t+1) = \begin{cases} p^{(i)}(t) & \text{if } \Delta c^{(i)} < 0 \text{ and } p^{(i)}(t) = g, \\ p^{(i)}(t) & \text{if } \Delta c^{(i)} > 0 \text{ and } p^{(i)}(t) = -g, \\ \text{rand}\{-1, 1\} & \text{otherwise.} \end{cases} \quad (6)$$

In other words, if i loses capital by playing the preferred game (as decided upon using the decision rule), i will take the risk by sticking to the same preference. Furthermore, if i gains capital despite not playing the preferred game, i will continue with the risk by not switching preference. For the other cases, i will continue to randomise its preference.

Lastly, individuals $i \in N$ with $\beta \in [14/24, 1]$ are set as *"Reckless"* individuals. Reckless individuals adopt an irrational strategy while at the same time giving greater weight to its own preference. Thus, the aggregation rule given by Eq. (1) also applies for i. It also uses the universal decision rule and the feedback is

$$p^{(i)}(t+1) = \begin{cases} p^{(i)}(t) & \text{if } \Delta c^{(i)} < 0 \text{ and } p^{(i)}(t) = g, \\ p^{(i)}(t) & \text{if } \Delta c^{(i)} > 0 \text{ and } p^{(i)}(t) = -g, \\ -p^{(i)}(t) & \text{if } \Delta c^{(i)} < 0 \text{ and } p^{(i)}(t) = -g, \\ -p^{(i)}(t) & \text{if } \Delta c^{(i)} > 0 \text{ and } p^{(i)}(t) = g. \end{cases} \quad (7)$$

The first two scenarios are similar to risk-taking. However, the "reckless" goes further by considering choices that are not favorable. If i loses capital by playing the game opposite to its preference, i will be reckless and switches to the game preference that resulted in the loss. Furthermore, if i gains capital by playing the game preferred, i will continue to be reckless by deviating from the game that resulted in the gain.

We investigate this risk-taking preference aggregation game on two networks—a scale-free random network and a social network. We used the `NetworkX 2.5` [36] package available with Python for the creation, manipulation, and investigation of the dynamics of Parrondo's games when applied to complex networks. In this article, we set the bias of Parrondo's games to be $\epsilon = 0.005$. The scale-free network is generated using the Barabási-Albert algorithm [37], with the following properties: size of node set (number of players) $|N| = 4039$, size of edge set (number of connections) $|E| = 88374$ and average degree $\langle k \rangle = 43.7603$, this is achieved by setting the built-in random graph properties `barabasi_albert_graph(4039,22)`. These properties were chosen to match a real-world Facebook social network with the following properties: $|N| = 4039$, $|E| = 88234$ and $\langle k \rangle = 43.6910$. This social network is obtained from [38]. All other graph properties are given in Table 1, with degree distribution shown in Fig. 1.

For each network, set each individual node's initial capital $c^{(i)}(0) = 0$. This can be carried out in general because we are only interested in the change of capital with time t. The other variables β_i and $p_i(t)$ are assigned random values drawn uniformly from their respective domains. We investigate the expected capital of all individuals $\langle C(t) \rangle$ as a function of discrete time t to show Parrondo's paradox and the individual capitals $c^{(i)}(t = 100)$ as a function of β to show Simpson's paradox. A total of 10^6 simulations were performed and averaged over.

Table 1
Dataset statistics of the BA scale-free network and the social media network.

Properties	BA Network	Social Network
Nodes	4039	4039
Edges	88,374	88234
Average clustering coefficient	0.03792	0.6055
Average degree	43.7603	43.6910
Number of triangles	87,511	16,12010
Diameter	4	8

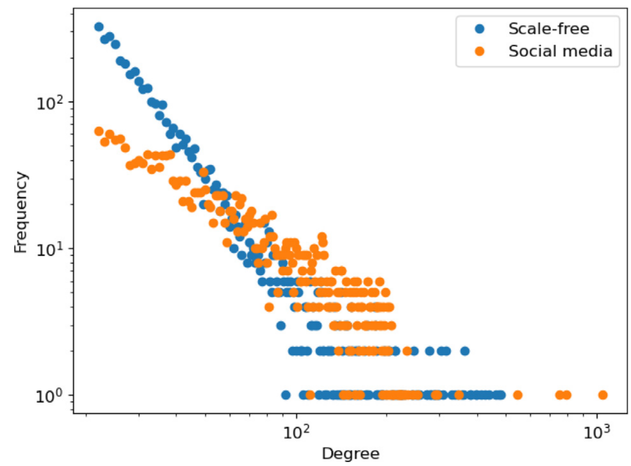
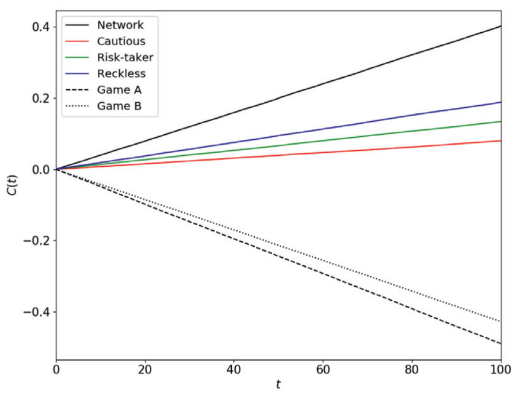


Fig. 1. Degree distribution of BA scale-free network defined by `barabasi_albert_graph(4039,22)`, and the real-world Facebook social network from Ref [38].

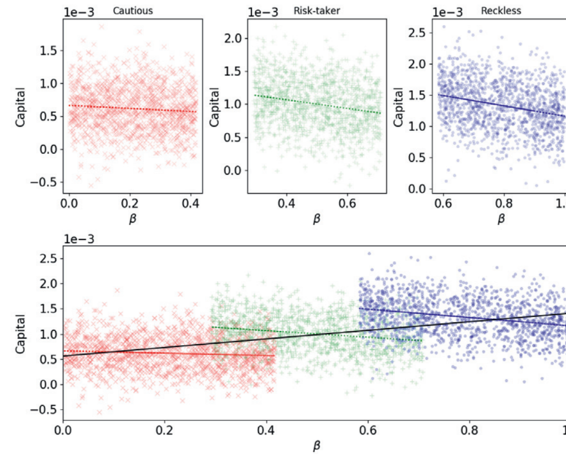
3. Results & discussion

Under the set up described, we observe Parrondo's paradox for the scale-free network (see Fig. 2a) and social media network (see Fig. 3a). While games A and B are individually losing when played by individuals in the network, a combination of the two games through preference aggregation leads to a winning outcome regardless of risk-taking behaviour. The expected capital at $t = 100$ for each of the different cases are reported in Table 2. Importantly, this study also uncovers the emergence of Simpson's paradox, at the same time, Parrondo's paradox is also being observed. Individually, each risk-taking group have decreasing capital with increasing β. However, when put together, both the scale-free network (see Fig. 2b) and social media network (see Fig. 3b) reveal Simpson's paradox. The properties are reported in Table 2.

We further provide analysis on how each of the average degree and average clustering coefficient of a scale-free network affects the ratios between the linear regression of each partition to the gradient of the linear regression of the entire network of size $N = 100$ in Fig. 4. In all cases, both Parrondo's and Simpson's paradox are observed. Furthermore, the strength of Simpson's paradox, determined by comparing the ratio of gradients between each partition and the network, reveal no significant correlation to the average degree nor the average clustering coefficient. A similar experiment was conducted on a Holme-Kim clustering scale-free network [39,40]. By setting the number of nodes and average degree of the network to be similar to that of the social media network, we are able to change the average clustering coefficient by invoking `powerlaw_cluster_graph(4039, 22, p)`, and altering p, which is the probability of forming a triangle, thus varying the average clustering coefficient. We similarly observe in Fig. 5 that when the average clustering coefficient is independently altered, no significant correlation can be found between the ratio of gradients between each partition and the network.

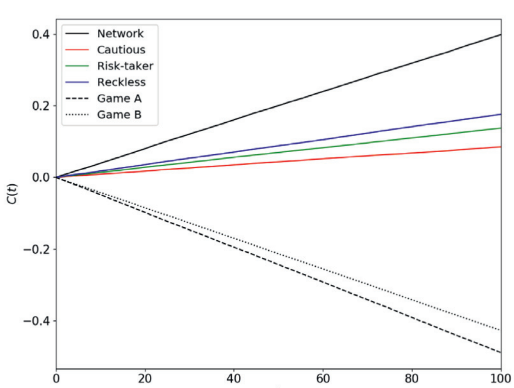


(a) Expected capital $\langle C(t)\rangle$ against time for a scale-free undirected network. Note that both games A and B are losing (decreasing with time). For all cases, the Parrondo's effect is observed for which two losing games combine to give a winning outcome. The labels "Cautious", "Risk-taker" and "Reckless" describe the expected capital gains of each type of risk-taking behaviour. The label "Network" is the total expected capital gain of the entire network.

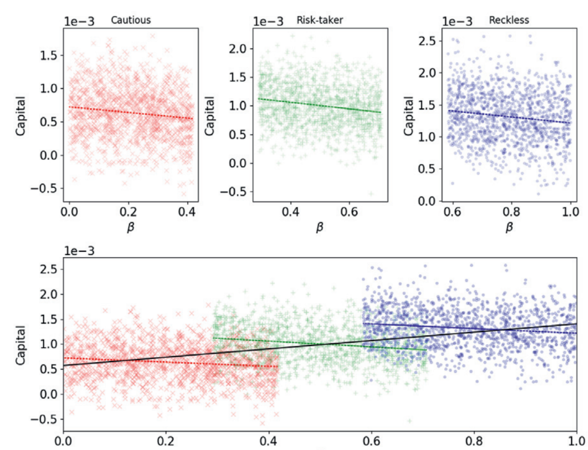


(b) Capital of player at $t = 100$ against β for a scale-free undirected network. The value of β characterises the risk-taking behaviour of players in the network. In each risk-taking category, the individual capital decreases with increasing β. The trend is reversed when collectively considered. The values of the gradients are reported in Table 2.

Fig. 2. Emergence of Parrondo's paradox and Simpson's paradox in a scale-free network.



(a) Expected capital $\langle C(t)\rangle$ against time for a Facebook undirected network.



(b) Capital of player at $t = 100$ against β for a Facebook undirected network. The values of the gradients are reported in Table 2.

Fig. 3. Emergence of Parrondo's paradox and Simpson's paradox in a social media network.

Table 2
Summary table of the expected capital per player at $t = 100$, $\langle C\rangle$ and the relationship m between the individual capitals at $t = 100$ and β for both the scale-free and social media networks. The total number of simulations per discrete time is 10^6. $\langle C\rangle$ is the average capital, r is the correlation coefficient, and m is the gradient of the regression line.

	Scale-free Network				Social media Network			
	Network	Cautious	Risk-taker	Reckless	Network	Cautious	Risk-taker	Reckless
$\langle C\rangle$	0.40144	0.07968	0.13366	0.18811	0.40144	0.07968	0.13366	0.18811
r	0.48199	−0.07928	−0.21437	−0.26464	0.46458	−0.13407	−0.18928	−0.14204
$m \times 10^{-4}$	8.5055	−2.2939	−6.5609	−8.2790	8.3252	−4.2086	−5.7517	−4.6975

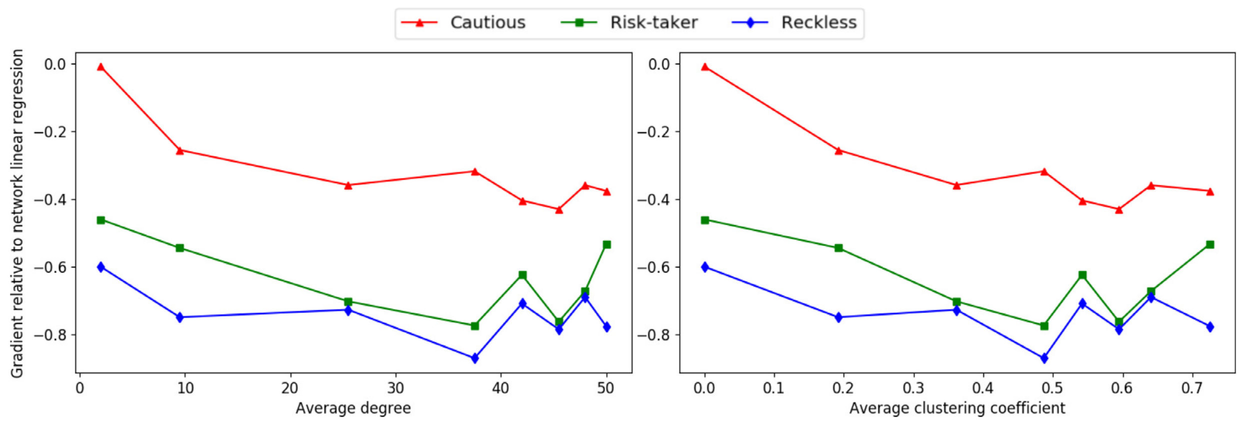


Fig. 4. Graphs showing the inductive analysis of how the average degree and average clustering coefficient of a scale-free network, for $N = 100$, each affects the ratios between the partition gradient and the network gradient. Each data point generated by taking 100 networks of the same parameters, and averaging over 10^5 preference aggregation Parrondo's games.

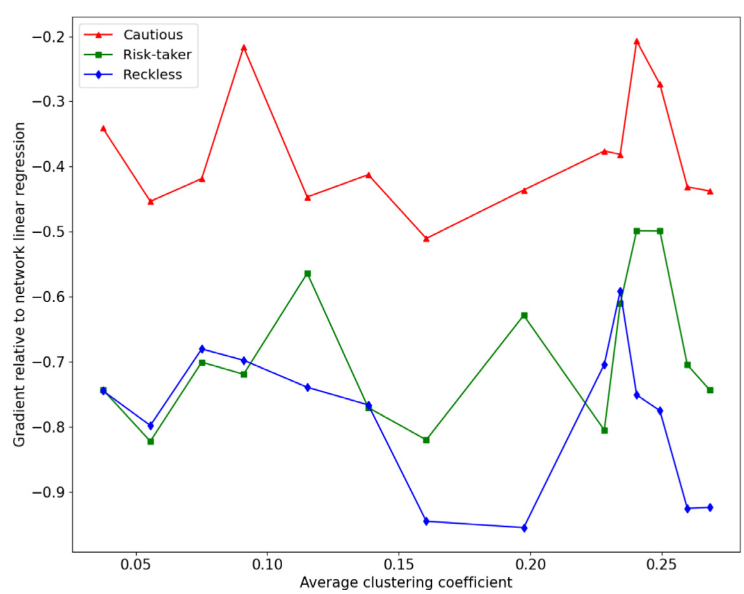


Fig. 5. Graphs showing the inductive analysis of how the average clustering coefficient of a Holme-Kim clustering scale-free network, for $N = 4039$ and average degree of 43.7, each affects the ratios between the partition gradient and the network gradient. Each data point generated by averaging over 10^6 preference aggregation Parrondo's games.

Our results in here suggest that the emergence of Simpson's paradox is largely due to the social rules governing the playing of Parrondo's games (more than the intrinsic structure of the network). Finally, we conjecture that Parrondo's games played on a partitioned network, using the preference aggregation framework described, can result in the emergence of Simpson's paradox. A comparison to a tag-based aggregate rule [31] reveal that in the latter (refer to supplementary information), Simpson's paradox does not emerge. To investigate how other aggregate rules such as the tag-based mechanism amalgamated with individual node connections, influence the emergence of Parrondo's paradox and Simpson's paradox, motivates future work.

4. Conclusion

Both Parrondo's paradox and Simpson's paradox have the interesting property of reversing trends. This work builds on social networks and reveal the emergence of both paradoxes and could inspire further research to uncover such instances in other social settings. Through the use of preference aggregation and modelling risk-taking behaviours when playing Parrondo's games, we show that not only is it possible for Parrondo's effect to emerge in the entire network, the same effect is also seen in each individual risk-taking behaviour groups. Beyond that, by assigning behaviours, we are also able to examine the trend between these behaviour factors and the individual capital gains. Through our simulations, Simpson's paradox is revealed. Moreover, an analysis of the strength of Simpson's paradox as a function of the average degree and clustering coefficient of a theoretical random network reveal that this result is not coincidental. This allows us to conjecture that out model does indeed lead to a double paradox.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Joel Weijia Lai: Conceptualization, Software, Methodology, Validation, Formal analysis, Investigation, Data curation, Writing –

original draft, Writing – review & editing, Visualization. **Kang Hao Cheong:** Conceptualization, Methodology, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration, Funding acquisition.

Acknowledgments

This project was supported by Singapore Ministry of Education (MOE) Academic Research Fund (AcRF) Tier 2 (Grant No. MOET2EP50120-0021).

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.chaos.2022.111911.

References

[1] Cheong KH, Saakian DB, Zadourian R. Allison mixture and the two-envelope problem. Phys Rev E 2017;96(6):062303. doi:10.1103/physreve.96.062303.
[2] Cheong KH, Koh JM, Jones MC. Multicellular survival as a consequence of Parrondo's paradox. Proc Natl Acad Sci 2018;115(23):E5258–9. doi:10.1073/pnas.1806485115.
[3] Tan Z-X, Koh JM, Koonin EV, Cheong KH. Predator dormancy is a stable adaptive strategy due to Parrondo's paradox. Adv Sci 2020;7(3):1901559. doi:10.1002/advs.201901559.
[4] Cheong KH, Wen T, Lai JW. Relieving cost of epidemic by Parrondo's paradox: a COVID-19 case study. Adv Sci 2020;7(24):2002324.
[5] Lai JW, Cheong KH. Parrondo's paradox from classical to quantum: a review. Nonlinear Dyn 2020a;100(1):849–61. doi:10.1007/s11071-020-05496-8.
[6] Rajendran J, Benjamin C. Playing a true Parrondo's game with a three-state coin on a quantum walk. Europhys Lett 2018a;122(4):40004.
[7] Rajendran J, Benjamin C. Implementing Parrondo's paradox with two-coin quantum walks. R Soc Open Sci 2018b;5(2):171599.
[8] Lai JW, Cheong KH. Chaotic switching for quantum coin Parrondo's games with application to encryption. Phys Rev Res 2021;3(2):L022019. doi:10.1103/physrevresearch.3.l022019.
[9] Porfiri M, Jeter R, Belykh I. Antiresonance in switched systems with only unstable modes. Phys Rev Res 2021;3(2). doi:10.1103/physrevresearch.3.l022001.
[10] Mendoza SA, Peacock-López E. Switching induced oscillations in discrete one-dimensional systems. Chaos Solitons Fractals 2018;115:35–44. doi:10.1016/j.chaos.2018.08.001.
[11] Mendoza SA, Matt EW, Guimarães-Blandón DR, Peacock-López E. Parrondo's paradox or chaos control in discrete two-dimensional dynamic systems. Chaos Solitons Fractals 2018;106:86–93. doi:10.1016/j.chaos.2017.11.011.
[12] Rosas A. Synchronization induced by alternation of dynamics. Chaos Solitons Fractals 2021;153:111461. doi:10.1016/j.chaos.2021.111461.
[13] Ma HF, Cheung KW, Lui GC, Wu D, Szeto KY. Effect of information exchange in a social network on investment. Comput Econ 2017;54(4):1491–503. doi:10.1007/s10614-017-9723-3.
[14] Lai JW, Chang J, Ang L, Cheong KH. Multi-level information fusion to alleviate network congestion. Inf Fusion 2020;63:248–55. doi:10.1016/j.inffus.2020.06.006.
[15] Ye Y, Hang XR, Koh JM, Miszczak JA, Cheong KH, gang Xie N. Passive network evolution promotes group welfare in complex networks. Chaos Solitons Fractals 2020;130:109464. doi:10.1016/j.chaos.2019.109464.
[16] Zhang M, Wang S-Y, Hu X-T, Alfaro-Bittner K. Local and global information affect cooperation in networked Prisoner's dilemma games. Chaos Solitons Fractals 2021;150:111183. doi:10.1016/j.chaos.2021.111183.
[17] Lai JW, Cheong KH. Social dynamics and Parrondo's paradox: a narrative review. Nonlinear Dyn 2020b;101(1):1–20. doi:10.1007/s11071-020-05738-9.
[18] Tuma N. Social dynamics: models and methods. Oxford: Elsevier Science; 1984. ISBN 9780323156905
[19] Lazer DMJ, Pentland A, Watts DJ, Aral S, Athey S, Contractor N, Freelon D, Gonzalez-Bailon S, King G, Margetts H, Nelson A, Salganik MJ, Strohmaier M, Vespignani A, Wagner C. Computational social science: obstacles and opportunities. Science 2020;369(6507):1060–2. doi:10.1126/science.aaz8170.
[20] Harmer GP, Abbott D. Losing strategies can win by Parrondo's paradox. Nature 1999;402(6764):864. doi:10.1038/47220.
[21] Koh JM, Cheong KH. Generalized solutions of Parrondo's games. Adv Sci 2020;7(24):2001126.
[22] Wu D, Szeto KY. Extended Parrondo's game and Brownian ratchets: strong and weak Parrondo effect. Phys Rev E 2014;89(2):022142.
[23] Simpson EH. The interpretation of interaction in contingency tables. J R Stat Soc Ser B 1951;13(2):238–41. doi:10.1111/j.2517-6161.1951.tb00088.x.
[24] Wagner CH. Simpson's paradox in real life. Am Stat 1982;36(1):46. doi:10.2307/2684093.
[25] Kock N, Gaskins L. Simpson's paradox, moderation and the emergence of quadratic relationships in path models: an information systems illustration. Int J Appl Nonlinear Sci 2016;2(3):200. doi:10.1504/ijans.2016.077025.
[26] Krueger N, Dickson PR. How believing in ourselves increases risk taking: perceived self-efficacy and opportunity recognition. Decis Sci 1994;25(3):385–400. doi:10.1111/j.1540-5915.1994.tb01849.x.
[27] Nicholson N, Soane E, Fenton-O'Creevy M, Willman P. Personality and domain-specific risk taking. J Risk Res 2005;8(2):157–76. doi:10.1080/1366987032000123856.
[28] Zinn JO. The meaning of risk-taking – key concepts and dimensions. J Risk Res 2017;22(1):1–15. doi:10.1080/13669877.2017.1351465.
[29] Libby R, Fishburn PC. Behavioral models of risk taking in business decisions: a survey and evaluation. J Account Res 1977;15(2):272. doi:10.2307/2490353.
[30] Reyna VF, Rivers SE. Current theories of risk and rational decision making. Dev Rev 2008;28(1):1–11. doi:10.1016/j.dr.2008.01.002.
[31] Riolo RL, Cohen MD, Axelrod R. Evolution of cooperation without reciprocity. Nature 2001;414(6862):441–3. doi:10.1038/35106555.
[32] Stoner JA. Risky and cautious shifts in group decisions: the influence of widely held values. J Exp Soc Psychol 1968;4(4):442–59. doi:10.1016/0022-1031(68)90069-3.
[33] Jagau S, Offerman T. Defaults, normative anchors, and the occurrence of risky and cautious shifts. J Risk Uncertain 2018;56(3):211–36. doi:10.1007/s11166-018-9282-6.
[34] Edwards W. The theory of decision making. Psychol Bull 1954;51(4):380–417. doi:10.1037/h0053870.
[35] Mukherjee N, Zabala A, Huge J, Nyumba TO, Esmail BA, Sutherland WJ. Comparison of techniques for eliciting views and judgements in decision-making. Methods Ecol Evol 2018;9(1):54–63. doi:10.1111/2041-210x.12940.
[36] Hagberg AA, Schult DA, Swart PJ. Exploring network structure, dynamics, and function using networkX. In: Varoquaux G, Vaught T, Millman J, editors. Proceedings of the 7th python in science conference. Pasadena, CA USA; 2008. p. 11–15.
[37] Albert R, Barabási A-L. Statistical mechanics of complex networks. Rev Mod Phys 2002;74(1):47–97. doi:10.1103/revmodphys.74.47.
[38] Leskovec J, Mcauley J. Learning to discover social circles in ego networks. In: Pereira F, Burges CJC, Bottou L, Weinberger KQ, editors. Advances in neural information processing systems, vol. 25. Curran Associates, Inc.; 2012. p. 539–47.
[39] Holme P, Kim BJ. Growing scale-free networks with tunable clustering. Phys Rev E 2002;65(2). doi:10.1103/physreve.65.026107.
[40] Rong Z, Yang H-X, Wang W-X. Feedback reciprocity mechanism promotes the cooperation of highly clustered scale-free networks. Phys Rev E 2010;82(4). doi:10.1103/physreve.82.047101.