Information Fusion 63 (2020) 248–255

Contents lists available at ScienceDirect

Information Fusion

journal homepage: www.elsevier.com/locate/inffus

# Multi-level information fusion to alleviate network congestion

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ARTICLE INFO

*Keywords:*
Information flow
Fusion process
Urban flow
Traffic network
Social dynamics
Ising model

ABSTRACT

While increasing urban traffic can be an indicator of development, this inevitably results in traffic congestion in urban road networks. Is there a way to manage traffic flow through the control of traffic signals such that the overall network congestion is improved? Traffic light signals can be represented as two states of an Ising model. It is possible for traffic lights to "communicate" with each other through a fusion process from a remote management control system. This requires collection of information which can be fed to a centralized decision-making control mechanism. We first explore the fusion process between traffic signals and show that it is possible for traffic flow in a city to follow the phase transition as exhibited in the 2D Ising model. The model will be extended to show that a random switching between signalling control mechanisms can result in congested traffic being susceptible to transit out of congestion.

## 1. Introduction

The management of traffic flow requires real-time information and prediction to respond to potential congestion [1]. Road infrastructure geometry, interconnectivity and relative timings of traffic lights at intersections will also impact the severity of congestion. The information collected about the traffic conditions can be fed to a centralized decision-making control mechanism [2,3]. The information, then undergoes a fusion process, where a positive feedback is relayed back to the traffic signal grid, and this allows for ease of traffic congestion, a simplified model is described in Fig. 1. The primary purpose of traffic signals in a road network is to regulate traffic flow, especially in response to congestion during peak hours. From the psychosocial perspective, most drivers tend to utilize their own self-interest instead of all road users' [4–8]. German mathematician Dietrich Braess observes that adding one or more roads to an existing network may end up impeding overall traffic flow – commonly known as Braess's paradox [9–11]. The reason for such a phenomenon taking place is because individual wishes to optimize their own utility rather than considering the group utility. This is analogous to finding a Nash equilibrium in a multi-player game, where the Nash equilibrium does not necessarily give the best possible result for any player [12]. We have proposed an original and novel fusion process of easing traffic congestion by performing utility optimization on present road infrastructure by considering group interest instead of optimizing the travel time for individuals.

Statistical mechanics has been used to explore evolutionary games in the thermodynamic limit [13,14], with extension to exploring the effects of the thermodynamic limit in quantum games [15,16]. It is observed that cooperation among individuals exist even when defection should be a choice for every player. The players were modelled as interacting agents using concepts from statistical mechanics, in particular, the Ising spin model. The Ising model was originally used in statistical mechanics to model interacting magnetic spins to show phase transition [17–20]. The two-dimensional (2D) square-lattice model of a two-level state is the simplest model to simulate phase transition. The 2D system is typically used to simulate the effects that temperature has on the magnetization of a material. A 2D lattice of magnetic dipoles at fixed lattice sites are allowed to interact with each other. These dipoles can either take an "up" state or "down" state and flip according to its interaction with its nearest neighbours. As temperature changes, the probability of flipping changes. Thus, it is possible for a material to lose its magnetization at some critical temperature, a process we call phase transition. The same idea can be employed in traffic network with interacting parts. Statistical mechanics is useful as a means to investigate traffic flow by considering the "microscopic" models of vehicular traffic and interaction between vehicles resulting in "macroscopic" physical phenomena [21].

Information fusion is key to managing road congestion and traffic flow, for instance, with machine learning techniques [22,23] used to observe and predict traffic evolution [24]. Information from traffic signals and the associated congestion is collected and fed to an external control mechanism that eases the congestion across the traffic grid. By

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https://doi.org/10.1016/j.inffus.2020.06.006
Received 24 February 2020; Received in revised form 26 May 2020; Accepted 14 June 2020
Available online 21 June 2020
1566-2535/

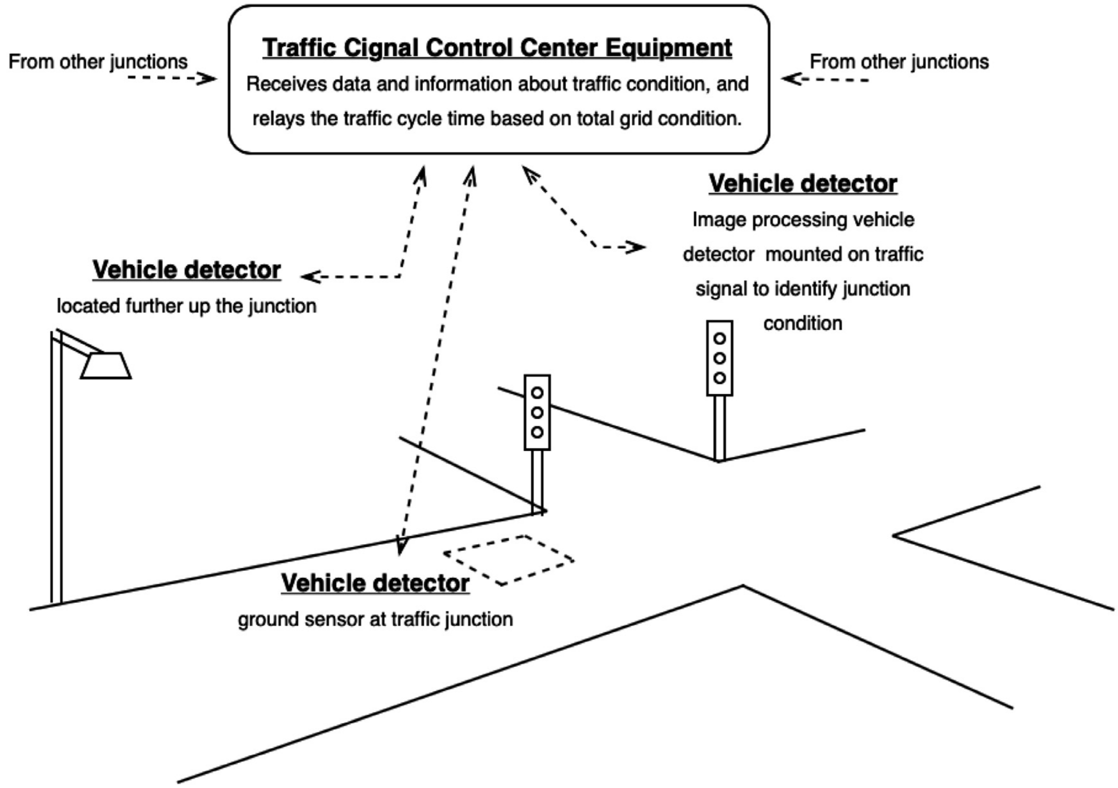


**Fig. 1.** Simple schematic of how sensors can be used along and at traffic junctions as data collection instruments, which feeds the information back to a centralized control system. This centralize control system decides on an appropriate cycle time, based on the fusion of information, then relays positive feedback to the traffic signals.

considering the interaction between neighbouring roads and signals, the time-discrete evolution of a road network can be simulated to shed light on the factors contributing to the phase transition from free-traffic to congestion and vice-versa [25–27].

We first ask a fundamental question: is it possible to perform switching of traffic control mechanisms to decrease the net total waiting time at all traffic signals in a 2D road network? Consider two losing games being combined in a random or periodic order to give a winning outcome, this is known as Parrondo's paradox [28]. Inspired by the flashing Brownian ratchet, Parrondo's paradox has found applications in physics [29–33], biology [34–37], engineering [38,39], social dynamics [40] and economics. A family of Parrondo's games that considers an ensemble of players and applies *social* rules in which the probabilities of the games are defined by the state of its neighbours is called cooperative Parrondo's games. The game has a "winning" outcome if the collective average capital of all the players is improved [41,42]. In prior work, cooperative traffic management has been introduced to achieve more optimal traffic network performance [43,44] using a game-theoretic approach [45]. However, these solutions cover cooperative dynamic routing between traffic management, drivers and machine-to-machine infrastructure; and may not be practical to be implemented in real-world scenarios.

In this paper, an original and novel method comprising the Ising spin model and cooperative games [46,47] is being proposed. Within our proposed method, the coupling constant in an Ising model is determined by a cooperative switching mechanism, simulated using a cooperative Parrondo's game. We then use the numerical results of this amalgamated framework to predict its effects on the phase transition observed in a traffic network. Our work here reduces the complexity of a traffic network by modelling it as a 2D cooperative system in relation to the interaction dynamics between traffic control management and machine-to-machine infrastructure. Our work also represents the first study to propose a solution via cooperative Parrondo's games based on the Ising spin model. Section 2 introduces the Ising spin model and explains how the parameters in the model correspond to the analogous features of a traffic network (see also Table 1). Next, Section 3 introduces the traffic control mechanism modelled after Parrondo's games, which controls the behavior of the coupling constant in the Ising model. Finally, we discuss the important implications of our results in Section 4.

## 2. Ising model

To set up the 2D Ising model in our transportation context, we first note that there are two possible states at each traffic intersection, represented by each lattice site. In our model, we assume that traffic signals are local and its corresponding traffic flow is affected only by its nearest neighbour. Interactions between neighbouring traffic conditions are naturally taken into account by the traffic flows between them. Only nearest neighbours need to be considered, as interaction beyond nearest neighbouring junctions does not immediately affect the state at the junction. Consider a simplistic one-dimensional case in the form of a single road with multiple sequential interactions (traffic signals) in Fig. 2. Suppose there is congestion on the road caused by a red signal on $S_2$, then the signal at $S_1$ is irrelevant for traffic flow beyond $S_2$. Similarly, the congestion may affect the congestion at traffic signal $S_3$. This can be modelled as a 1D Ising model with two possible states and analytically solved using methods in statistical mechanics [48]. This linear symmetry can then be extended to a 2D road network by applying an orthogonal rotation symmetry at each lattice site.

Consider a set $\Lambda$ of lattice sites, forming a two-dimensional $N \times N$ square lattice, with periodic boundary conditions. For each lattice site $k \in \Lambda$, there is a discrete variable $\sigma_k$ such that $\sigma_k \in \{+1, -1\}$, representing the site's state where $+1$ represents the "up" state and $-1$ represents the "down" state. A state configuration, $\sigma = (\sigma_k)_{k\in\Lambda}$ is an assignment of state value to each lattice site. For any two adjacent sites $i, j \in \Lambda$, there

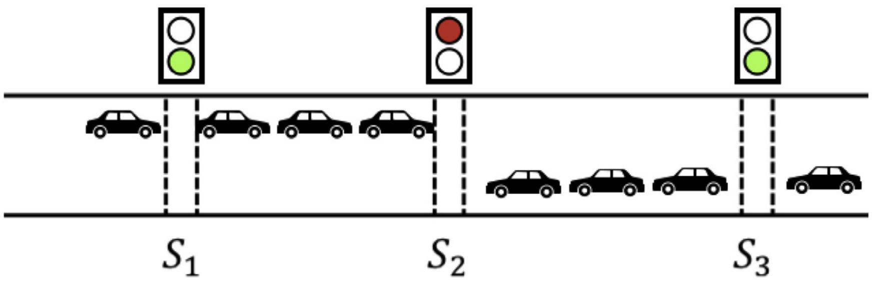


**Fig. 2.** Congestion is only affected by the state of neighbouring traffic signals. This can be extended to an entire 1D stretch of road by applying translational symmetry.

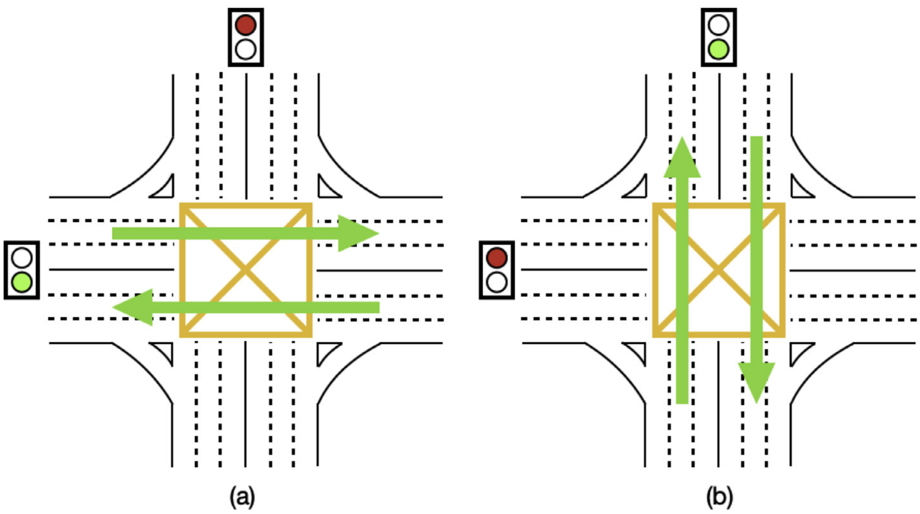


**Fig. 3.** Two states of a traffic junction representative of a two-state system at each lattice site of an Ising Model. (a) represents an "up" state, which corresponds to traffic in the north/south direction and (b) is the "down" state, corresponding to east/west traffic.

is an interaction $J_{ij}$. The value $J_{ij}$ is related to the probability of vehicles making a left or right turn at each junction, hence forming a coupling constant which connects each site with its neighbours.

An external agent is able to monitor all traffic junctions and then decide with some probability, in discrete time, whether to flip a traffic light from "up" state, which corresponds to traffic in the north/south direction and the "down" state, corresponding to east/west traffic. The states are described in Fig. 3. This external intervention is able to monitor the "energy" of the whole network. We treat this "energy" as a physical quantity dependent on a configuration $\sigma$, which can be described by a Hamiltonian

$$\mathcal{H}(\sigma) = -\sum_{\langle i,j \rangle} J_{ij}\sigma_i\sigma_j \tag{1}$$

where $\langle i, j\rangle$ is the sum over nearest neighbours. For simplicity, we shall assume that the pairwise interaction at each lattice site is probabilistic according to a signal control scheme based on the state of the neighbouring traffic signal, i.e, $J_{ij} = J(\sigma)$ for all pairs $i, j \in \Lambda$.

We now consider two different signal control schemes, for some random number $r \in [0, 1)$:

(A) Independent of neighbouring states,

$$J(\sigma) = \begin{cases} 1 & \text{if } r \le p \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

(B) Dependent on number of neighbouring states in "up" configuration,

$$J(\sigma) = \begin{cases} 1 & \text{if } r \le p_i \\ 0 & \text{otherwise} \end{cases}, \tag{3}$$

where $i$ is the number of neighbouring states in the "up" configuration.

The choice of $J(\sigma)$ is analogous to the probability of "winning" or "losing" in Parrondo's games. The dynamics of choosing $J(\sigma)$ is also presented in Section 3, in the context of cooperative Parrondo's games. The choice of $p_i \in \{p, p0, \dots, p4\}$ will be presented in Section 3.1. Our analytic derivations below justify our approach as it shows that the dynamics of the traffic network problem does not deviate from the Ising model, only the choice of $J(\sigma)$ will determine the outcome.

**Table 1**
Parameters from the Ising model and its corresponding interpretation in a traffic network.

| Ising Model parameter | Traffic Network Interpretation |
|---|---|
| $\Lambda$ | Traffic network. A set of lattice sites describing the topology of the traffic network |
| $\sigma$ | State of traffic signal. The flow of traffic north-south (+1) or east-west (−1). |
| $J$ | Coupling constant. Determined by the cooperative management mechanism, related to the probability of vehicles not heading straight at each junction. |
| $m$ | Congestion index. It gives a macro perspective of the scale of congestion of the whole traffic network. $0 \le m \le 1$. |
| $\chi$ | Susceptibility. The susceptibility of the traffic network transitioning from congestion to free. |
| $\mathcal{H}$ | "Energy" purely determined by the states $\sigma$ and $J$. It gives a perspective of the evolution (with time) of the flow of the traffic. |
| $\xi$ | Associated with the degrees of freedom of the lattice $\Lambda$. |
| $\langle \cdot \rangle$ | Average of a physical quantity over $N$ simulations. |

Let $A_\sigma$ be an observable of the system for some state $\sigma$. For a system in equilibrium, the average value of the observable is given by

$$\langle A \rangle = \frac{1}{\mathcal{Z}} \sum_\sigma A_\sigma e^{-\mathcal{H}_\int / k_B \xi}, \tag{4}$$

where $\mathcal{Z}$ is the partition function $\mathcal{Z} = \sum_\sigma e^{-\mathcal{H}_\int / k_B \xi}$, and we denote "energy" of the system as $\mathcal{H}_\sigma$ for configuration $\sigma$.

The Monte Carlo simulation method allows us to determine $\langle A \rangle$. In our case, we want to find $\langle m \rangle$. We first consider a Markov chain as a discrete process which starts from an arbitrary configuration $\mu$ at time $t = 0$ and generates a new configuration at time $t + \Delta t$. Each transition depends only on the current state of the system and not on the previous history (Markov property). This transition is governed by a transition probability $P(\mu \to \nu) \geq 0$, satisfying

$$\sum_\nu P(\mu \to \nu) = 1. \tag{5}$$

The system at any given time is characterized by the probabilities $p_\mu$ of being in the state $\mu$. We can write the probability of finding the system in a state (at a later time) $\nu$ as:

$$p_\nu(t + \Delta t) = \sum_\mu p_\mu(t) P(\mu \to \nu), \tag{6}$$

which can be written as a vector equation $p(t + \Delta t) = P p(t)$, where the matrix $P$ has elements $P_{\mu\nu} = P(\mu \to \nu)$. Note that in general $P(\mu \to \nu) \neq P(\nu \to \mu)$.

Let us derive the case for which this converges to the equilibrium of the system, that is

$$\omega_\mu = \frac{1}{\mathcal{Z}} \exp(-\mathcal{H}_\mu / k_B \xi). \tag{7}$$

Therefore,

$$\omega_\nu = \sum_\mu \omega_\mu P(\mu \to \nu). \tag{8}$$

Using Equation (5) this condition can be written as

$$\sum_\mu \omega_\nu P(\nu \to \mu) = \sum_\mu \omega_\mu P(\mu \to \nu), \tag{9}$$

which implies

$$\omega_\nu P(\nu \to \mu) = \omega_\mu P(\mu \to \nu), \tag{10}$$

this is the *detailed balance* condition. By substituting the condition into Equation (7) to (10), we obtain the transition probability:

$$\frac{P(\nu \to \mu)}{P(\mu \to \nu)} = e^{-(\mathcal{H}_\mu - \mathcal{H}_\nu)/k_B \xi}. \tag{11}$$

Following the Metropolis algorithm for detailed balance, the external agent will decide to flip the state at a randomly chosen lattice site $k$ with the following probability:

$$\mathcal{P}(\sigma, \tau) = \begin{cases} e^{-d\mathcal{H}/\xi} & \text{if } d\mathcal{H} > 0, \text{where } d\mathcal{H} = \mathcal{H}_\sigma - \mathcal{H}_\tau \\ 1 & \text{otherwise} \end{cases} \tag{12}$$

where we have set $k_B = 1$, a constant with appropriate dimensions. Lastly, we shall define the physical quantity analogous to the magnetization of the Ising spin system as

$$m = \frac{1}{N} \sum_i \sigma_i. \tag{13}$$

We refer to $m$ in the context of the traffic network as the "congestion index". $m$ gives information about the congestion of the entire traffic network. It is a measure of the congestion of a traffic network, the macroscopic quantity. A good physical intuition is to think of $|m|$ as how the flow of traffic aligns. If $|m| \approx 1$, it implies that over time, the flow of traffic is only in the north-south direction or east-west direction. Vehicles in the other direction (east-west in the case when the flow is north-south) will not be moving. This will eventually lead to a congestion because vehicles will eventually turn (either left or right) into a congested junction. On the other extreme, when $|m| \approx 0$, then we have a balance of traffic flow in the north-south direction and the east-west direction. It is worth noting that this does not imply there is no local congestion within the larger network. When $0 < |m| < 1$, the state of the traffic is a mix between the two extreme cases. For larger values of $|m|$, it is analogous to having denser and larger domains of congestion across the network. As $|m|$ tends to zero, these domains become smaller and sparse.

## 3. Decision-making control mechanism

These cooperative Parrondo games are physically analogous to choosing a certain control mechanism to switch the lights from green to red and vice-versa. To model a cooperative game based on the Ising spin model, we consider performing probabilistic switching dynamics at each traffic signal according to Fig. 4.

Similar to the Ising model, only the traffic signal and traffic of adjacent intersections will affect each intersection, imposing periodic boundary conditions (in the case of large traffic grids, the road network is

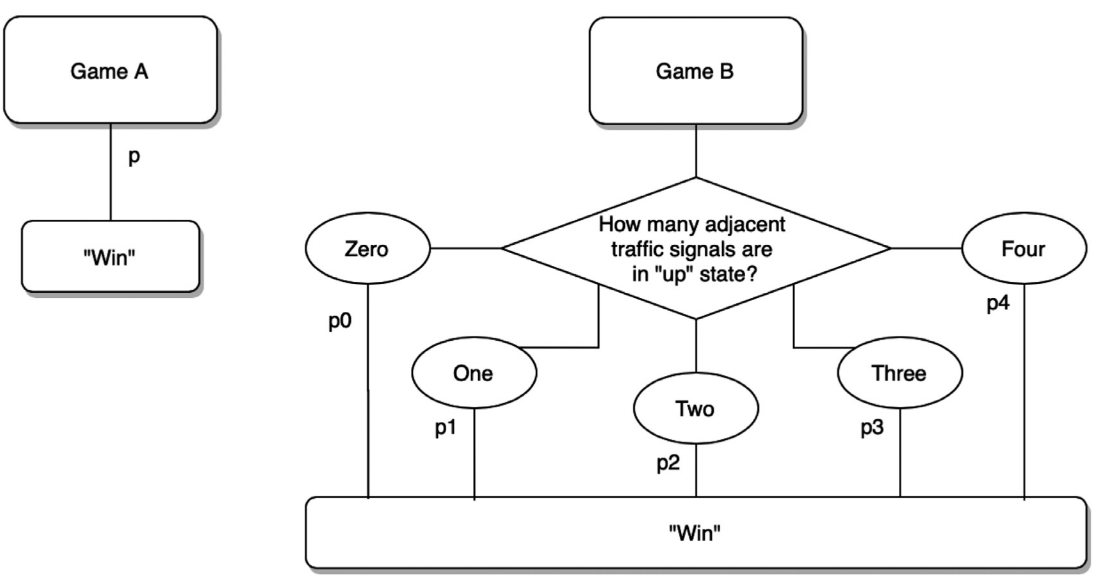


**Fig. 4.** Cooperative Parrondo's paradox scheme according to the traffic conditions.

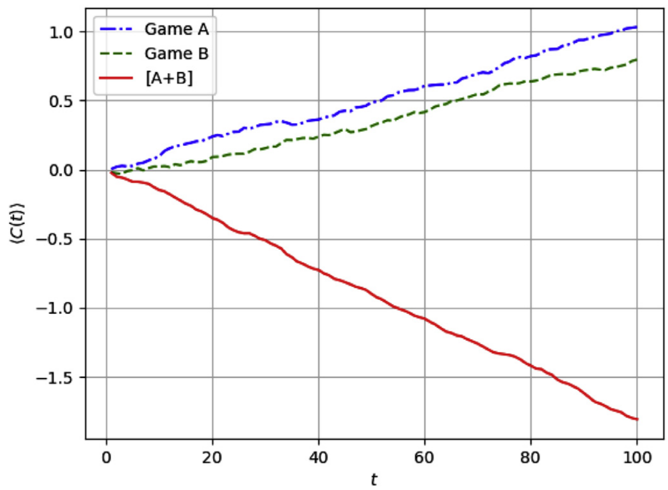


**Fig. 5.** Experimental results from implementing Fig. 4, with $N = 20$, $n = 10^6$ and $t = 100$, showing that mixing two "losing" control mechanism can give a "winning" control mechanism.

self-similar). By considering cooperation between nearest neighbours, we will next show that the game-theoretic Parrondo's paradox is applicable in our context.

### *3.1. Ising cooperative parrondo model*

To model our traffic network, we take a reasonable number of cross junctions $N = 20$. The simulation is then averaged over $n = 10^7$ iterations per simulation, with each simulation performed over $t = 10^5$ time steps. It is worth noting that "winning" is intuitively defined as spending less time (capital, $C(t)$) at an intersection. Therefore, in effect, we are showing that mixing two control mechanisms of positive capital gives a negative capital. The probabilities of obtaining a "losing" strategy (i.e $\langle C(t)\rangle > 0$) are $p = \frac{1}{2} + \varepsilon$, $p_0 = 1.0$, $p_1 = 0.9 + \varepsilon_1$, $p_2 = 0.2 + \varepsilon_2$, $p_3 = 0.1 + \varepsilon_3$ and $p_4 = \varepsilon_4$. As an illustration, we have used the following $\varepsilon$ values: $\varepsilon = 0.005, \varepsilon_1 = 0.05, \varepsilon_2 = 0.05$, $\varepsilon_3 = 0.01$ and $\varepsilon_4 = 0.01$.

Simulation results in Fig. 5 predict the winning control mechanism when there is random switching performed between mechanism A and B. Having shown that there exists a winning strategy by mixing the control mechanism, this is now applied to the coupling constant $J(\sigma)$ to show that it is indeed advantageous in mixing the control mechanism. The pseudocode of these simulations can be found in the Supplementary Information. We perform two simulations using micro-canonical Monte Carlo simulations [49]:

(1) Starting with free traffic (i.e randomized states at each lattice site), we iterate through values of $\xi$ for $t$ discrete time steps to find the critical value $\xi_0$ for which a phase transition takes place.
(2) Take two starting configurations:
 (i) Free traffic – randomized $\sigma$,
 (ii) Congestion – aligned $\sigma$
 for the mixed control mechanism and illustrate its evolution after $t$ discrete time steps for some determined value of $\xi$ close to $\xi_0$.

We next perform a series of simulation for finding the critical parameter $\xi_0$. This parameter is analogous to the temperature of a system as defined in statistical mechanics. Temperature is defined based on a system's degrees of freedom. In the case of traffic modeling, $\xi$ takes the same definition defined by the degrees of freedom of the state configuration $\sigma$ [50]. The analytic solution [51] for $\xi_0$ of a magnetic spin Ising model is found to be

$$\xi_0 = \frac{2}{\ln(1+\sqrt{2})} J \approx 2.269\, J, \tag{14}$$

for a constant $J$ implying that it is solely dependent on the coupling constant. Next, we define the physical property analogous to the "magnetic susceptibility" of the Ising model, which we refer to as simply "susceptibility".

$$\chi = \langle m^2\rangle - \langle m\rangle^2. \tag{15}$$

Our simulation results for control mechanisms A, B and $[A+B]$ are presented in Figs. 6, 7 and 8, respectively. The outcome of the "congestion index" shows very good agreement with that obtained from an Ising spin model. The predicted uncertainty is $O(10^{-1})$ with a linear rate of convergence.

We now focus on exploring how traffic evolves with time $t$, determined by discrete time step when starting from a free traffic state and a congestion state for a some value of $\xi$. As an illustration, we choose values of $\xi$ close to $\xi_0$, varying the value of $\xi = \{2.2, 2.3, 2.4, 2.5\}$, taking $t = 10^6$ time steps for each simulation. The simulation results are shown in Fig. 9.

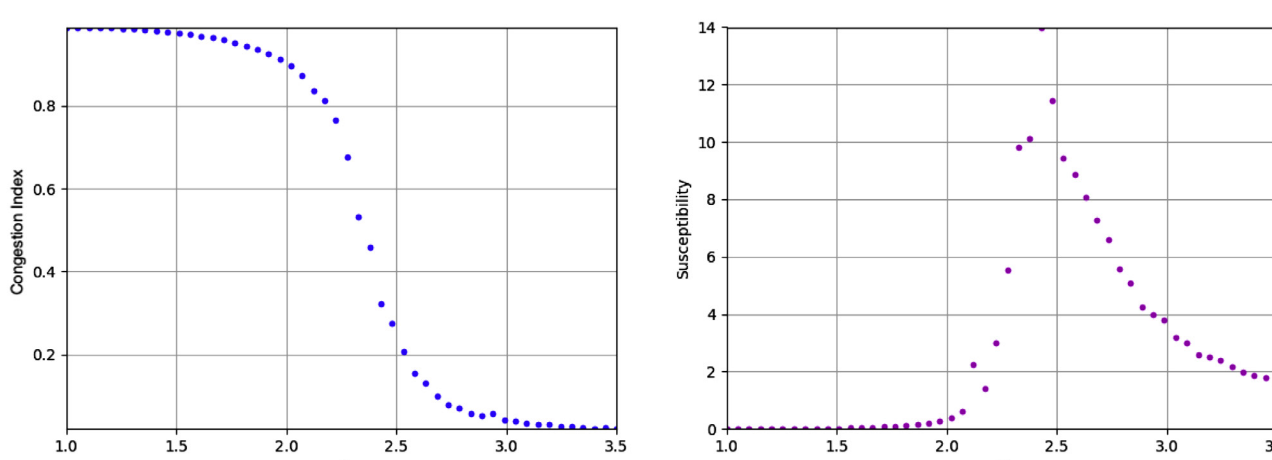


**Fig. 6.** Numerical results for the Ising model, where the coupling constant $J(\sigma)$ follows control mechanism A averaged over 100 simulations.

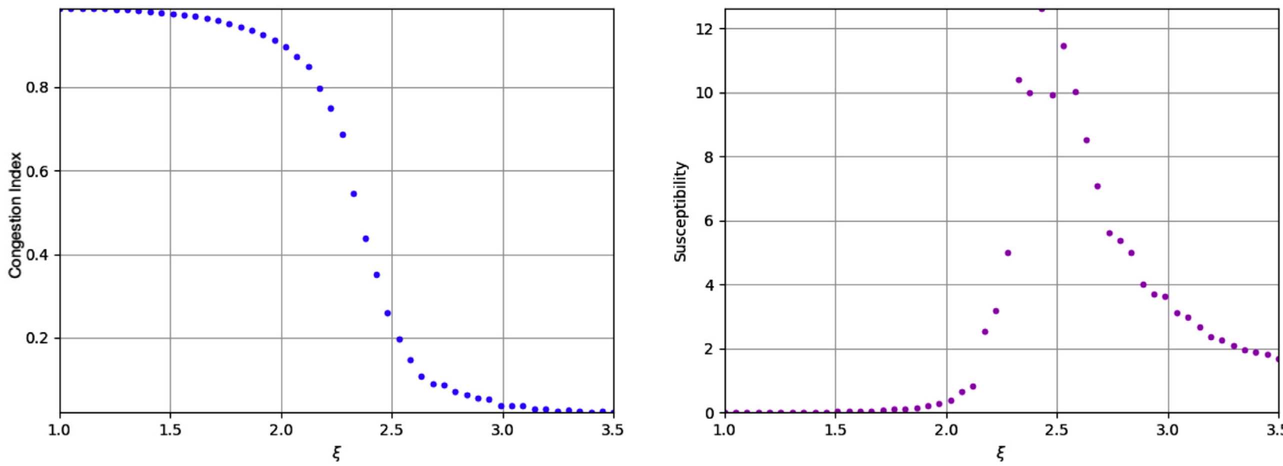


**Fig. 7.** Numerical results for the Ising model, where the coupling constant $J(\sigma)$ follows control mechanism B averaged over 100 simulations.

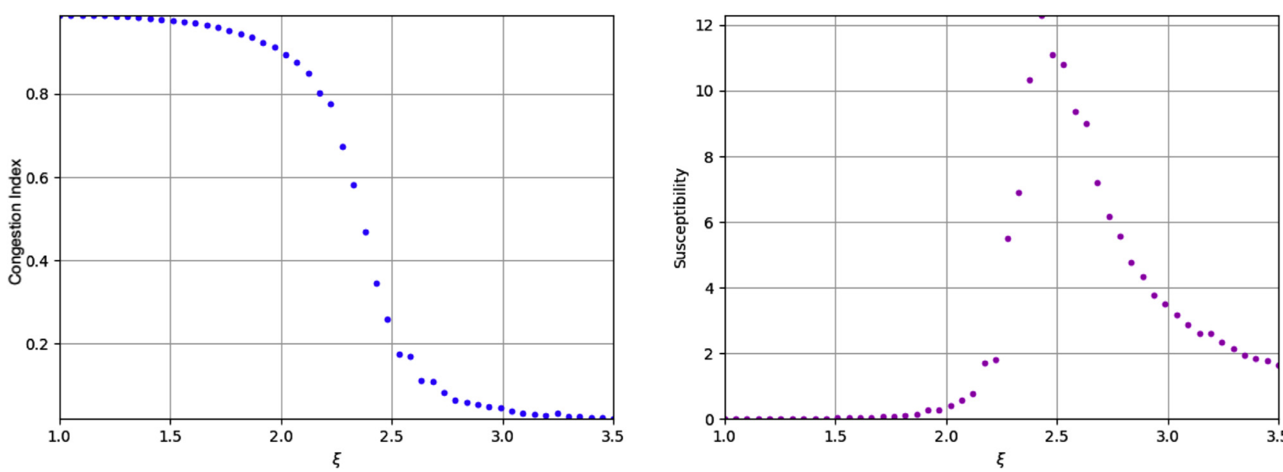


**Fig. 8.** Numerical results for the Ising model, where the coupling constant $J(\sigma)$ follows the mixed control mechanism $[A + B]$ averaged over 100 simulations.

## 4. Discussion

From our simulation results in Fig. 6 to 8, we were able to obtain the critical value $\xi_0 \approx 2.4$ by performing the Metropolis algorithm and Monte Carlo simulations for the 2D square-lattice Ising model. For all three control mechanisms, the critical value is the same. This is significant as this implies that the critical point of transition is independent of the choice of control mechanism. It is important to note that the congestion index $m$ decreases with increasing $\xi$ for all control mechanism, which is as expected. It is known that the coupling constant does affect the critical value $\xi_0$, as established by Onsager's exact solution [51]; the susceptibility at the critical value for the mixed control mechanism, $\chi_{[A+B]} = 12.4 \pm 0.1$, is found to be lower than in the case of following a single control mechanism with $\chi_A = 14.1 \pm 0.1$ and $\chi_B = 12.8 \pm 0.1$. Physically, this implies that the system is more susceptible to change state from "congestion" ($|m| = 1$) to "free traffic" ($|m| = 0$).

Our proposed Ising model has revealed that above the critical value $\xi_0$, the starting configuration of the traffic network does not affect the long-term effect of traffic flow. It has been shown that $\lim_{t\to\infty} |m| = 0$ for both the aligned and random configuration. However, locally, there remains domains of gridlocks as revealed in Fig. 9. This observation suggests that in the process of improving the overall network congestion, there will bound to be localized domains that continue to experience grid-lock, which cannot be alleviated simply by using the method of switching control mechanisms.This is in agreement with the physical intuition motivated in the introduction. The size of the domains is also sparse and localized. Importantly, we have shown the total network congestion can be improved.

Defining the physical quantities and fusion process that affect $\xi$ and methods to determine the critical value $\xi_0$ is crucial in studying traffic network congestion. Traffic management is a complex problem affected by many variables, including vehicle length, ideal acceleration and deceleration and safety distances, among others. To definitively determine $\xi$, each network has to be studied in greater details, and even so, the stochastic nature of traffic can only be simulated, as is performed by implementing the Ising model. Furthermore, this work focuses on two-state traffic flow at each junction. Traffic networks are complex systems that can be modelled using other complicated systems of traffic control which may or may not be better than our current model. However, such comparisons are not the focus of the current study but this motivates future work for which each direction of flow at a traffic junction can be independently considered as a state. Such consideration will give rise to a longer cycling time, but coordinated traffic flow. With four possible flow directions, there will be four states to consider, and a spin-3/2 system should be used then.

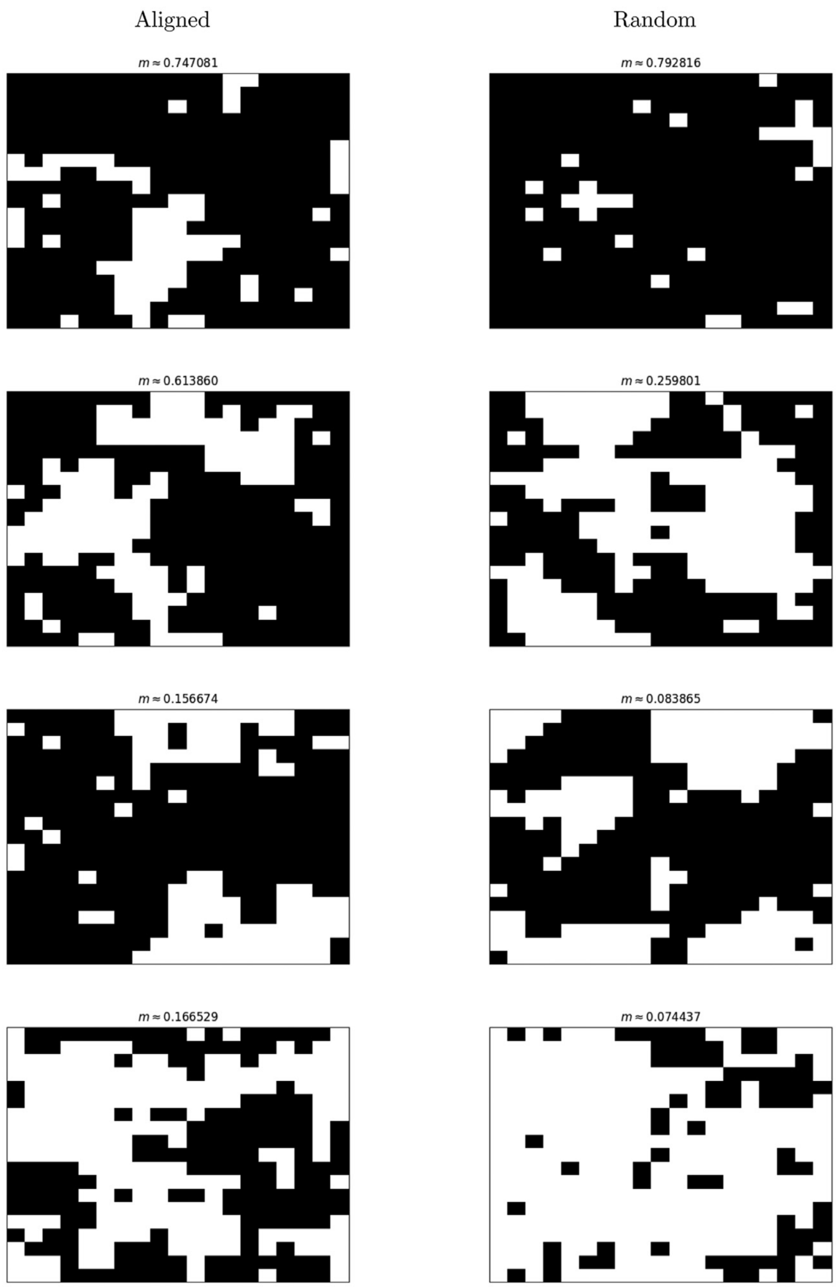


**Fig. 9.** Macroscopic experimental outcome after $t = 10^6$ time steps for coupling constant determined by the $[A + B]$ mixed control mechanism, for $N = 20$. White and black squares represent the "down" and "up" states, respectively. The first column corresponds to states initially starting with aligned "spin" states (congestion), while the second column corresponds to states initially starting with random "spin" states (free traffic). Each row corresponds to $\xi = \{2.2, 2.3, 2.4, 2.5\}$ respectively.

## 5. Conclusion

In conclusion, we have shown that traffic network in a transportation system can be modelled using a 2D Ising model with coupling constant $J(\sigma)$ determined by the dynamics of the cooperative Parrondo's paradox. This will result in the traffic network being more susceptible to a phase transit from "congestion" to "free traffic". Crucially, the study of factors that governs the parameter $\xi$, as well as history dependent networks motivates future work. Our work here represents a novel and fully implementable algorithm that can be used in real-life as part of city planning and traffic management by fusing the information through a means of data collection at each traffic junction. Reducing the effects of traffic gridlock will also minimize environmental pollution, thus making urban traffic more sustainable in the long run.

## Declaration of Competing Interest

The authors declare that they have no conflict of interest.

## CRediT authorship contribution statement

**Joel Weijia Lai:** Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Visualization, Writing - original draft, Writing - review & editing. **Jie Chang:** Formal analysis, Investigation, Visualization, Writing - review & editing. **L. K. Ang:** Formal analysis, Investigation, Visualization, Writing - review & editing. **Kang Hao Cheong:** Conceptualization, Methodology, Validation, Formal analysis, Investigation, Resources, Data curation, Visualization, Writing - original draft, Writing - review & editing, Supervision, Project administration, Funding acquisition.

## Acknowledgement

This work was supported by the Singapore University of Technology and Design Grant No. SGPCTRS1804 and SUTD-MIT International Design Centre Grant No. IDG21900101 and IDIN19001.

## Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.inffus.2020.06.006

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